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Fiscal Policy in a Growing Economy with Financial Frictions and Firm Heterogeneity

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Fiscal Policy in a Growing Economy with Financial Frictions and Firm Heterogeneity*

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Abstract

This paper constructs a tractable model of endogenous growth with financial frictions and firm heterogeneity. We introduce factor income tax, consumption tax as well as the government consumption into the base model and explore the growth effect of fiscal policy. We show that from the qualitative perspective, the long-run effects of fiscal actions in our model are similar to those obtained in the representative-agent models. However, the quantitative impacts of fiscal policy on long-run growth in our setting can be substantially different from those established in the model where agents are homogeneous and there is no financial friction.

JEL Classification: D31, O41

Keywords: fiscal policy, financial constraints, firm heterogeneity, endogenous growth

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1 Introduction

The effect of fiscal policy on long-term economic growth has been one of the central concerns in growth economics. Particularly, in the 1990s a number of authors investigated the role of fiscal policy in the context of endogenous growth models. A small sample includes Barro (1990), Devereux and Love (1994 and 1995), Futagami et al. (1993), Jones et al. (1997), Lucas (1990), Milesi-Feretti and Roubini (1998a and 1998b), Mino (1989 and 1996), Rebelo (1991) and Stokey and Rebelo (1995).[1] In the endogenous growth models, the long-term growth rate is sensitive to distortionary fiscal policy. Therefore, the literature in the 1990s focused on the growth effect of fiscal actions such as factor income taxation, consumption tax, public investment and government consumption. The general finding of this literature is that fiscal policy may yield decisive impacts on long-run growth of an economy.[2] Their findings demonstrate that differences in fiscal policy can be one of the relevant determinants of the cross-country divergence in growth performance. Such a conclusion is in stark contrast to the outcome of the neoclassical (exogenous) growth theory in which fiscal policy fails to affect the long-run growth rate of national income.

It is to be noted that although the foregoing investigations on fiscal policy and endogenous growth employ various types of models, they share the common features: all of the studies cited above assume that agents are homogeneous and financial markets are perfect. It is natural to guess that these restrictive assumptions are related to the notable growth effect of fiscal policy in endogenous growth models. To examine this point, the present paper reconsiders the long-run effects of fiscal policy in the presence of financial frictions and heterogeneity of firms. We construct a simple model of endogenous growth in which production efficiency of firms are heterogeneous and their investments are subject to financial constraints. Based on this model, we examine the growth effects of factor income tax, consumption tax as well as government consumption. Our main concern is to explore the differences between

[1] Devereux and Love (1994 and 1995), Lucas (1990), Milesi-Feretti and Roubini (1998a), Mino (1996) and Stokey and Rebelo (1995) examine the effects of factor income taxation in two-sector endogenous growth models with physical and human capital. Jones et al. (1997) study optimal taxation in a similar setting. Milesi-Feretti and Roubini (1998b) also discuss the growth effect of consumption tax. Barro (1990), Futagami et al. (1993) and Devereux and Love (1994 and 1995) examine models in which productive public spending sustains persistent growth.

[2] A notable exception is a two-sector endogenous growth model with physical and human capital examined by Lucas (1988). In his model the balanced growth rate is determined by the learning technology of the households alone and the rate of income tax fails to affect the long-run growth rate.

the policy impacts in our model and those in the corresponding representative-agent model without financial frictions.

More specifically, the baseline setting of our discussion is an AK growth model with variable labor supply. This model is highly tractable and it is probably the simplest framework for discussing endogenous growth with flexible labor supply. In addition, if endogenous growth is not allowed, the model reduces to the prototype model of business cycles that has been widely used in the real business cycle literature. As the foregoing studies confirm, the long-run effects of fiscal policy in this model mainly stem from its impact on the labor-leisure choice of the representative household. In this paper we assume that there are workers and entrepreneurs. The workers supply labor, consume and accumulate their financial assets. Each entrepreneur runs a firm whose production efficiency is different from each other. Moreover, the investment of each entrepreneur is subject to a borrowing constraint, which yields a cutoff level of production efficiency. The entrepreneurs whose efficiency levels are less than the cutoff give up production and become rentiers. It turns out that the cutoff level of production efficiency depends on the wealth distribution between the workers and entrepreneurs. Since fiscal policy affects the aggregate wealth distribution, in addition to the labor-leisure choice of workers, there is an additional effect of fiscal policy on long-term growth in our model economy.

Our paper presents two main findings. First, it is shown that from the qualitative view point, the growth effect of each type of fiscal policy in our model is essentially the same as that in the representative agent model: a rise in each rate of tax lowers the balanced-growth rate, whereas a higher income share of government consumption accelerates long-run growth. Second, we find that the quantitative impacts of fiscal policy in our setting would be different from those obtained in the representative-agent economy. Our numerical analysis shows that the growth effects of taxation on capital income and government consumption in our model economy tend be smaller than those established in the representative agent economy. On the other hand, the growth effects of taxation on wage income and consumption spending in both models are not significantly different from each other. Moreover, it is shown that a change in the rate of tax on profit income has a significant, negative impact in our model. Since the excess profit income does not exist in the competitive equilibrium in the representative agent economy, this finding is established only in the model with financial frictions and firm heterogeneity.

According to the existing empirical studies on fiscal policy and economic growth, we do not have enough evidences which support the theoretical outcomes of studies on fiscal policy and endogenous growth. For example, the cross-country studies by Easterly and Rebelo (1995) and Mendoza et al. (1997) find no significant correlation between long-run growth rate of real GDP and the average rate of tax. Additionally, Stokey and Rebelo (1995) argue that despite the several tax reforms in the United States after World War II, the real growth rate of the US economy has been fairly stable.[3] Since the growth effect of tax policy also depends on how the tax revenues are used, those empirical findings do not necessarily reject the results of theoretical studies that focus on the distorting effects of fiscal policy. Although this paper does not intend to resolve the discrepancy between theory and emprics, our study demonstrates that departing from the standard, representative agent setting would be useful to consider this issue from a broader perspective.

Related Literature

Because of its tractability, the AK growth model with endogenous labor-leisure choice has been frequently used in the literature. An earlier discussion on this type of model is given by Benhabib and Farmer (1994). Turnosky (2000) presents a detailed analysis of fiscal policy in the representative agent version of this framework[4]. Amano et al. (2009) and Amano and Itaya (2012) also discuss the effects of income tax in the AK growth model in which labor supply is endogenously determined.

Our formulation of financial frictions basically follows Kiyotaki and Moore (1997).[5] While there is a large body of literature on macroeconomic models with financial frictions, there are relatively small number of studies that consider both financial imperfection and firm heterogeneity. Among others, Moll (2014) introduces credit constraints on investment into a neoclassical growth model with heterogeneous firms and investigates the effect of financial frictions on the total factor productivity. He shows that the presence of financial constraints determines the cutoff level of efficiency of the firm, which affects the productivity of the

[3] See also Romer and Romer (2010).

[4] Turnovsky (1999) also discusses the effects of fiscal policy in a small-open economy version of the base model.

[5] See also Kiyotaki (1998).

aggregate economy. Using the same analytical framework, Itskhoki and Moll (2014) explore the optimal tax schemes in a small-open economy. Additionally, Liu and Wang (2014) construct a neoclassical growth model with financial constraints and firm heterogeneity in which firms' total production costs are subject to borrowing constraints. Their primary concern is to examine the presence of sunspot-driven business cycles in such an environment. Our formulation of the model with financial constraints on investment is close to that used by Moll (2014) and Itskhoki and Moll (2014).[6] The key difference is that our model sustains continuing growth and, hence, our study is an endogenous-growth counterpart of the existing research mentioned above. Mino (2015) also discusses an endogenous growth model with financial frictions and firm heterogeneity in which production technology is a simple AK type with fixed labor supply. Since our model allows endogenous labor-leisure choice, it is close to the standard real business cycle models. Chen and Mino (2014) consider alternative forms of financial constraints both in the exogenous and endogenous growth versions of our baseline framework.

It is to be pointed out that Jaimovich and Rebelo (2014) also examine the long-run impact of income taxation in an endogenous growth model with firm heterogeneity. They use an R&D based growth model in which the efficiency of developing new technology is assumed to be heterogeneous. These authors numerically reveal that the relation between rate of income tax and long-run growth rate can be nonlinear. Namely, a change in the rate of income tax yields little impact on growth as long as it does not take an extremely high value.[7] In their model, neither financial imperfection nor endogenous labor supply is considered. Also their model ignores capital accumulation. Although our model does not present an explicit non-linear relation between growth and taxation, the relatively small impacts of factor income taxes in our numerical evaluation are similar to the finding by Jaimovich and Rebelo (2014).

2 Model

2.1 Agents

Workers

[6] See slso Buera and Moll (2012) and Buera et al. (2015).

[7] Their research, therefore, tries to resolve the discrepancy between theory and evidence mentioned above.

There is a continuum of identical workers with a unit mass. The representative worker maximizes a sum of discounted utilities

$$U_w = \int_0^\infty e^{-\rho_w t} \left(\log C_w - \frac{N^{1+\gamma}}{1+\gamma} \right) dt, \qquad \rho_w > 0, \quad \gamma > 0$$

subject to the flow budget constraint

$$\dot{B} = (1 - \tau_r)\, rB + (1 - \tau_w)\, wN - (1 + \tau_c)\, \dot{C_w} - T. \tag{1}$$

In the above, C_w N B, w and r respectively denote consumption, labor supply, asset holding, real wage rate and the real interest rate. In addition, ρ_w denotes the worker's time discount rate, $\tau_r \in [0, 1)$ is the rate of tax on capital income, $\tau_w \in [0, 1)$ the rate of tax on labor income, τ_c (≥ 0) the rate of consumption tax, and T is a lump sum tax (a lump sum transfer if it has a negative value). The representative worker's optimal consumption-saving plan is also subject to the initial holding of asset, B_0, and the the non-Ponzi-game scheme such that

$$\lim_{t \to \infty} \exp \left(- \int_0^t (1 - \tau_r)\, r_s ds \right) B_t \geq 0.$$

The optimal levels of consumption, labor supply and asset holding should satisfy the following:

$$C_w N^\gamma = \frac{1 - \tau_w}{1 + \tau_c} w, \tag{2}$$

$$\dot{C}_w = C_w \left[(1 - \tau_r)\, r - \rho \right], \tag{3}$$

$$\lim_{t \to \infty} e^{-\rho_w t} B / C_w = 0. \tag{4}$$

Condition (2) means that the marginal rate of substitution of consumption for labor equals the tax-adjusted real wage rate. In addition, (3) is the Euler equation of optimal consumption and (4) is the transversality condition.

Entrepreneurs

Entrepreneurs also constitute a continuum with a unit measure. Each entrepreneur owns a firm. The production technology of firm owned by type i entrepreneur is

$$y_i = Q\left(z_i k_i\right)^{\alpha}\left(n_i K\right)^{1-\alpha}, \qquad Q > 0, \quad 0 < \alpha < 1, \qquad i \in [0,1], \tag{5}$$

where y_i, k_i and n_i are output, capital and labor of firm i, respectively. In the above, K denotes the aggregate level of capital in the economy at large. Following Romer (1986), we assume that the efficiency of labor input depends on the external effects of intangible capital which is proportional to the total stock of capital. Furthermore, it is assumed that the efficiency of capital denoted by z_i is heterogeneous among firms. The above specification shows that each firm has the same form of production technology except for the level of z_i. We assume that in each moment entrepreneurs draw capital efficiency z from a Pareto distribution whose cumulative distribution function is given by

$$F(z) = 1 - z^{-\psi}, \qquad \psi > 1. \tag{6}$$

Here, the shape parameter, ψ, expresses the degree of heterogeneity in production efficiency: a lower value of ψ means a higher level of heterogeneity in production technology among firms. We may interpret z_i as an idiosyncratic technological shock that hits each firm in each moment. In what follows, according to Liu and Wang (2013) and Itskhoki and Moll (2014), we assume that z is *iid* over time as well as across agents. As a result, owing to the law of large numbers, the population share of entrepreneurs who draw a particular level of z is stationary and deterministic.[8]

Each entrepreneur maximizes an expected sum of discounted utilities given by

$$U_{e,i} = E_0 \int_0^{\infty} e^{-\rho_e t} \log c_{e,i} dt, \qquad \rho_e > 0,$$

subject to

$$\dot{a}_i = (1-\tau_r)\, r a_i + (1-\tau_p)\, \pi_i - (1+\tau_c)\, c_{e,i}, \quad i \in [0,1] \tag{7}$$

where $c_{e,i}$ is consumption, a_i is stock of financial asset (net worth), π_i is a profit income and

[8] Our formulation is a simplified version of firm dynamics studied by, for example, Luttmer (2007 and 2010). Moll (2014) treats a more general case where shocks are persistent. Moll (2014) reveals that the qualitative results under such an extension are not substantially different from the outcomes in our simplified modelling.

$\tau_p \in [0, 1)$ denotes the rate of tax on profit income. We assume that the fiscal authority does not levy a lump sum tax on the entrepreneurs. In our model, the time discount rate of entrepreneurs, ρ_e, may be different from the workers' time discount rate, ρ_w. Each entreprenur also follows the no-Ponzi-game condition.

As a firm owner, the entrepreneur is subject to a financial constraint. It is assumed that the debt of an entrepreneur defined by $d_i = k_i - a_i$ should satisfy

$$d_i \leq \lambda a_i, \quad \lambda \geq 0.$$

This constraint means that each entrepreneur uses her net worth as a collateral. The above constraint is rewritten as

$$k_i \leq \theta a_i, \quad \theta = 1 + \lambda \geq 1. \tag{8}$$

That is, the capital stock held by an entrepreneur is restricted by its net worth. Or equivalently, the leverage ratio of the firm must be less than θ. If $\theta = +\infty$, then the financial market is perfect. In contrast, borrowing is not allowed, if $\theta = 1 \ (\lambda = 0)$.

We first formulate the entrepreneur's employment policy of labor and capital as a static optimization problem. In this paper it is assumed that households of workers and entrepreneurs own capital stock. As a producer, each entrepreneur employs labor and rents capital from the households. Thus, defining the before-tax excess profit of firm i as

$$\pi_i = [y_i - wn_i - (r + \delta)k_i],$$

we assume that the firm maximizes π_i by choosing n_i and k_i subject to the production technology (5) and the financial constraint (8). Note that the aggregate capital, K, in (5) is external to an individual firm, so that the firms take K as given when deciding their production plan. To derive the optimization conditions, it is helpful to note that an interior solution with respect to n_i gives

$$(1 - \alpha) \frac{y_i}{n_i} = w. \tag{9}$$

Using this condition, the excess profit is written as

$$\pi_i = \left[z_i \alpha Q \left(\frac{w}{(1-\alpha) QK} \right)^{\frac{\alpha - q}{\alpha}} - (r+\delta) \right] k_i,$$

implying that the firm selects k_i to maximize the above subject to $0 \leq k_i \leq \theta k_i$. Thus the optimal level of capital satisfies

$$\begin{aligned} k_i &= \theta a_i \text{ for } z_i > z^*, \\ k_i &= 0 \quad \text{for } z < z^*, \end{aligned}$$

where the cutoff level of z is given by

$$z^* = \frac{r+\delta}{\alpha Q} \left[\frac{w}{(1-\alpha) QK} \right]^{\frac{1-\alpha}{\alpha}}. \tag{10}$$

Consequently, the entrepreneurs who draw $z_i \geq z^*$ earn non-negative profits, while those who draw $z_i < z^*$ obtain negative profits. We assume that the entrepreneurs with $z_i \geq z^*$ participate in production activities. The rest of the entrepreneurs give up production and become rentiers. In what follows, we assume that the financial constraints always bind the active entrepreneurs.

Let us express the excess profit, π_i, in such a way that

$$\begin{aligned} \pi_i &= \left(\alpha \frac{y_i}{k_i} - (r+\delta) \right) k_i = \left\{ z_i \alpha \left[\frac{w}{(1-\alpha) Q} \right]^{\frac{\alpha-1}{\alpha}} K^{\frac{1-\alpha}{\alpha}} - (r+\delta) \right\} k_i \\ &= \hat{\pi}(z_i, r, w, K) \theta a_i, \end{aligned}$$

where we use the effective financial constraint, $k_i = \theta a_i$. In the above, $\hat{\pi}(z_i, r.w, K)$ represents the (before-tax) excess rate of return to capital received by the active entrepreneurs. Therefore, the intertemporal consumption-saving plan of an active entrepreneur is to maximize U_i^e subject to

$$\dot{a}_i = [(1-\tau_r) r + (1-\tau_p) \hat{\pi}(z_i, r, w, K) \theta] a_i - (1+\tau_c) c_{e,i},$$

where

$$\hat{\pi}(z_i, r, w, K) = z_i \alpha \left[\frac{w}{(1-\alpha) Q} \right]^{\frac{\alpha-1}{\alpha}} K^{\frac{1-\alpha}{\alpha}} - (r+\delta).$$

Let us denote the value function of an entrepreneur at time t by $v_i\left(a_{i,t}, z_i\right)$. Then the Bellman equation for this problem is set as follows:[9]

$$\rho_e v_{i.t}\left(a_{i,t}, z_i\right) = \max_{c_{e,i}} \left\{\log c_{e,i,t} + \frac{1}{dt} E_t dv_i\left(a_{i,t},, z_i\right)\right\},$$

where $a_{i.t}$. changes according to

$$da_{i,t} = \left[\left(1-\tau_r\right) r_t a_{i,t} + \left(1-\tau_p\right) \hat{\pi}\left(z_i.r_t, w_t, K_t\right) \theta a_{i,t} - \left(1+\tau_c\right) c_{e,i,t}\right] dt.$$

Following Itskhoki and Moll's (2014) discussion, we may confirm that as long as z_i is *iid,* the optimal consumption of an active entrepreneur is given by $\left(1+\tau_c\right) c_{e,i} = \rho_e a_{i}.$[10] Since the inactive entrepreneurs' flow budget constraint is $da_i = \left(1-\tau_r\right) r a_i dt - \left(1+\tau_c\right) c_i dt$, their optimal consumption is also satisfy $\left(1+\tau_c\right) c_i = \rho_e a_i$. Heince , the optimal consumption function of each entrepreneur is:

$$c_{e,i} = \frac{\rho_e}{1+\tau_c} a_i, \quad \text{for all } i \in [0,1]. \tag{11}$$

Government

We assume that the fiscal authority balances its budget in each moment by adjusting the lump sum tax , T. Hence, the government's flow budget constant is given by

$$G = \tau_r r (A+B) + \tau_w w N + \tau_p (Y - wN - (\delta + r) K) + T,$$

where A $\left(= \int_0^1 a_i di\right)$ denotes the total asset holding of entrepreneurs.[11] We also assume that

[9] Note that the value function is not stationary because it involves r_t and w_t. When $dt \to 0$, this equation becomes the Hamilton-Jacobi equation, that is, the continuous-time counterpart of the Bellman equation.

[10] Itskhoki and Moll (2014) assume that the value function takes a form of $v_t\left(a_{i,t}, z\right) = M \log a_{it} + \mu \chi_t(z)$,where M and μ are undetermined constants. This specification yields $E_t dv_t\left(a_{i,t}, z\right) = M\left(da_{i,t}/a_{i,t}\right) + \mu E_t d\chi_t(z)$ .Then using the flow budget constraint, the Bellman equation in our model is written as

$$\begin{aligned} & \rho_e M \chi_t(z) + \rho_e M \log a_{i,t} \\ = \quad & \max_{c_{i,t}} \left\{\log c_{e,i,t} + \frac{M}{a_{i,t}}\left[r_t + \hat{\pi}\left(a_{i,t}, z_i\right) \lambda a_{i,t} - \left(1+\tau_c\right) c_{e,i,t}\right] + \mu \frac{1}{dt} E_t d\chi_t(z)\right\} \end{aligned}$$

Based on the guess and verify approach, it is shown that $\mu = \rho_e$ and, hence, the first-order condition, $1/c_{e,i,t} = \left(1+\tau_c\right) \mu / a_{it}$, leads to $c_{i,t} = \rho_e a_{i,t}.\left(1-\tau_c\right)$.

[11] We introduce the lump-sum tax (or subsidy), T, to keep intratemporal budget balance of the government. An alternative formulation is to assume that there is no lump sum tax and the government spending G is

the government consumption is proportional to the total income in such a way that

$$G = \eta Y, \quad \eta \in [0, 1).$$

Note that the aggregate income is distributed to capital, labor and excess profits so that $Y = (r + \delta) K + wN + \Pi$, where Π is the aggregate excess profits [12] As a result, the government budget balance is rewritten as

$$G = \tau_r r (A + B) + \tau_w wN + \tau_p \Pi + T. \tag{12}$$

2.2 Market Equilibrium Conditions

Final Goods Market

The aggregate demand for final goods consists of private consumption, investment and government consumption. Thus the market equilibrium condition for final good is:

$$Y = C_w + C_e + \dot{K} + \delta K + G, \tag{13}$$

where $C_e \left(= \int_0^1 c_{e,i} di\right)$ is the total consumption of the entrepreneurs.

Financial Market

As mentioned before, there is a cutoff level of production efficiency, z^* in our economy. Entrepreneurs who draw z_i which is higher than z^* produce and they are subject to the financial constraints. The other entrepreneurs who draw $z_i < z^*$ become lenders. Thus the lenders in our economy are workers and inactive entrepreneurs, while the active entrepreneurs are borrows. The financial market equilibrium is described by

$$A + B = K. \tag{14}$$

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determined by the total tax revenue of the government. Under such a policy rule, the market equilibrium condition for final goods becomes

$$\dot{K} = (1 - \tau_r) rK + (1 - \tau_w) wN + (1 - \tau_p) \Pi - (1 - \tau_c) (C_w + C_e) - \delta K.$$

Thus changes in tax rates directly affect capital accumulation. Our formulation is helpful to focus on the distortionary effects of factor income tax on the agents' decision making, It is also useful to consider the effect of a change in government consumption under a given levels of tax rates.

[12] As for aggregation of k_i, n_i, y_i and π_i, see Footnote 13.

Therefore, if $B > 0$, both workers and entrepreneurs hold assets, whereas workers owe a debt if $B < 0$.[13]

3 Equilibrium Dynamics and the Balanced-Growth Path

3.1 Aggregation

Aggregate Production Function

Remember that we have assumed that production efficiency, z, is *iid* over time as well as across agents, so that distribution of a and z among entrepreneurs are independent each other in each moment. Also, notice that all the capital stock is employed by the entrepreneurs who draw $z_i \geq z^*$ and that those active entrepreneurs are subject to the financial constraints $k_i = \theta a_i$ Hence, the aggregated levels of capital and net worth satisfy the following relation:[14]

$$K = \theta \int_0^1 \int_{z \geq z^*} a_i F'(z)\, dz di = \theta A z^{*-\psi},$$

which presents an alternative representation of the cutoff level:

$$z^* = \left(\frac{\theta A}{K} \right)^{\frac{1}{\psi}}. \tag{15}$$

Equation (15) reveals that given parameter values θ and ψ, a rise in the relative wealth held by the entrepreneurs, A/K, increases the cutoff level of production efficiency.

Substituting (5) into (9) and aggregating it over i and z, we obtain the following equation:[15]

$$\frac{\psi}{1-\psi} z^{*(1-\psi)} = \left[\frac{w}{(1-\alpha) QK} \right]^{\frac{1}{\alpha}} z^{*-\psi} N. \tag{16}$$

[13] In our formulation, each entrepreneur is characterized by its asset holding, a, and production efficiency, z. Thus if we denote the joint distribution function of (a, z) by $\Gamma(a, z)$, the aggregate levels of capital, hours worked, output and profit income are repetitively defined as follows:

$$\begin{aligned} K &= \int k(a,z)\, d\Gamma(a,z), \qquad N = \int n(a,z)\, d\Gamma(a,z), \\ Y &= Q \int (zk(a,z))^{\alpha} (n(a,z) K)^{1-\alpha} d\Gamma(a,z), \quad \Pi = \int \pi(a,z)\, d\Gamma(a,z). \end{aligned}$$

[14] Given our assumption that z_i is *iid* , the general expressions of aggregate variables given in Footnote 13 can be expressed below.

[15] To drive (16), we use the relation, $\int_{z \geq z^*} z dF(z) = \frac{\psi}{1-\psi} z^{*(1-\psi)}$.

Aggregation of (9) presents

$$w = (1-\alpha)\frac{Y}{N}, \tag{17}$$

which states that the real wage equals the aggregate marginal productivity of labor. Using (16) and (17) , we find that the aggregate production function is expressed as

$$Y = Q\left(\frac{\psi}{\psi-1}z^*\right)^{\alpha} KN^{1-\alpha}. \tag{18}$$

It is to be pointed out that the average productivity of the firms whose production efficiency is higher than z^* is given by $\int_{z \geq z^*} z^{*\psi} z dF(z) = \frac{\psi}{\psi-1}z^*$. Hence, the above expression means that TFP of the aggregate technology depends positively on the average productivity of the active firms. In addition, substituting (15) into (18) , we see that the aggregate production function is also expressed as

$$Y = Q\left(\frac{\psi}{\psi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\psi}} \left(\frac{A}{K}\right)^{\frac{\alpha}{\psi}} KN^{1-\alpha}, \tag{19}$$

implying that under a given level of A/K, the aggregate output is linearly related to K.

Notice that if there is no firm heterogeneity so that $\psi = \infty$, then (19) becomes

$$Y = QKN^{1-\alpha},$$

which gives the aggregate production function with homogeneous firms. In the presence of financial frictions and firm heterogeneity, the term $\left(\frac{\psi}{\psi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\psi}} \left(\frac{A}{K}\right)^{\frac{\alpha}{\psi}}$ expresses the efficient wedge of the aggregate technology. Given parameter values of α, ψ and θ, the efficiency wedge becomes higher, as the asset share of the entrepreneurs, A/K, increases. Additionally, under a given level of A/K, the efficiency wedge is higher, either if the degree of firm heterogeneity is larger (ψ is smaller) or if the financial constraint is weaker (θ is larger) .

Asset Accumulation of Entrepreneurs

Inserting (10) into (18) and using (17) , we obtain

$$r + \delta = \frac{\psi-1}{\psi}\alpha\frac{Y}{K}. \tag{20}$$

If there is no firm heterogeneity ($\psi = +\infty$), then $r + \delta = \alpha Y/K$ so that the gross rate of return to capital equals the marginal product of the aggregate capital. In the presence of firm heterogeneity and financial frictions, $(\psi - 1)/\psi$ represents an investment wedge. Since $(\psi - 1)/\psi$ is lowered as ψ decreases, a higher degree of heterogeneity of firms raise the investment wedge.

Using $Y = (r + \delta)K + wN + \Pi,$ (17) and (20), we obtain the following:

$$\Pi = \frac{\alpha}{\psi} Y. \tag{21}$$

The above represents the aggregate excess profits earned by active entrepreneurs who participate in production activities. Equations $Y = rK + wN + \Pi$, (17) and (20) demonstrate that in the presence of financial constraint and firm heterogeneity, the aggregate non-wage income, αY, is divided into the rental income of capital and the excess profits.

Finally, (11) means that the aggregate consumption of the entrepreneurs is described by

$$C_e = \int_0^1 \frac{\rho_e}{1 + \tau_c} a_i di = \frac{\rho_e}{1 + \tau_c} A. \tag{22}$$

and aggregating entrepreneurs' flow budget constraint (7) gives

$$\dot{A} = (1 - \tau_r) rA + (1 - \tau_p) \Pi - \rho_e A. \tag{23}$$

This equation represents dynamics of the aggregate net worth in the economy at large.

Notice that using the budget constraints of workers (1), entrepreneurs (23) and the government (12), together with the equilibrium condition of financial market (14), we obtain the equilibrium condition for final goods given by (13).

3.2 Dynamic System

In view of (20), (21) , (22) and (23), we find that the aggregate behavior of entrepreneurs' net worth follows

$$\dot{A} = (1 - \tau_r) \left[\alpha \frac{\psi - 1}{\psi} \frac{Y}{K} - \delta \right] A + (1 - \tau_p) \frac{\alpha}{\psi} Y - \rho_e A. \tag{24}$$

By use of (1), (17) and (20), the workers' flow budget constraint is written as

$$\dot{B} = (1-\tau_r)\left[\alpha\frac{\psi-1}{\psi}\frac{Y}{K} - \delta\right]B + (1-\tau_w)(1-\alpha)Y - (1+\tau_c)C_w - T.$$

Our model involves three stock variables, K, A and B, but the financial market equilibrium condition (14) gives $B = K - A$. Thus we restrict of our attention to the dynamic motions of A and K in the following analysis.

From (2), (17) and (19) we obtain:

$$C_w N^{\gamma} = \frac{1-\tau_w}{1+\tau_c}\left(\frac{\psi}{\psi-1}\right)^{\alpha}\left(\frac{\theta A}{K}\right)^{\frac{\alpha}{\psi}}(1-\alpha)QKN^{-\alpha}. \tag{25}$$

If there is neither financial constraint nor firm heterogeneity, the above condition is reduced to $C_w N^{\gamma} = \frac{1-\tau_w}{1+\tau_c}(1-\alpha)QKN^{1-\alpha}$. That is, the marginal rate of substitution of consumption for labor equals the tax adjusted marginal product of labor. Thus the term $\frac{1-\tau_w}{1+\tau_c}\left(\frac{\psi}{\psi-1}\right)^{\alpha}\left(\frac{\theta A}{K}\right)^{\frac{\alpha}{\psi}}$ in (25) expresses the labor wedge beteen the marginal rate of substitution and the marginal product of labor.

From the above equation, the equilibrium level of total hours worked is:

$$N = \left[\frac{1-\tau_w}{1+\tau_c}\left(\frac{\psi}{\psi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\psi}}(1-\alpha)Q\right]^{\frac{1}{\alpha+\gamma}}\left(\frac{A}{K}\right)^{\frac{\alpha}{\psi(\alpha+\gamma)}}\left(\frac{C_w}{K}\right)^{-\frac{1}{\alpha+\gamma}}. \tag{26}$$

This equation shows that the equilibrium level of labor input increases with the entrepreneurs' relative wealth holding, A/K, and decreases with the workers' consumption relative to capital, C_w/K. The first effect on the equilibrium level of labor reflects the fact that a higher A/K raises the cutoff level z^*, which increases the productivity of the aggregate technology. Substituting (26) into (19) yields

$$Y = \Lambda\left(\frac{A}{K}\right)^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}\left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}}K, \tag{27}$$

where

$$\Lambda = \left(\frac{\psi}{\psi-1}\right)^{\frac{\alpha(1+\gamma)}{\alpha+\gamma}}\left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}}\theta^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}}Q^{\frac{1+\gamma}{\alpha+\gamma}}.$$

Equation (27) demonstrates that the equilibrium output still keeps an AK property: the aggregate output is proportional to the aggregate capital. Therefore, in the balanced growth equilibrium where A, K and C_w change at a common rate, the aggregate level of output-capital ratio, Y/K, stays constant as well. However, in the transition, the endogenous variables A/K and C_w/K change over time, so that the growth dynamics of our economy are more complex than that of the standard AK growth model where the economy always stays on the balanced growth path.

In sum, using (3), (13), (24) aand (27), we obtain a complete dynamic system with respect to K, A and C_w in the following manner:

$$\frac{\dot{K}}{K} = (1-\eta)\,\Lambda\left(\frac{A}{K}\right)^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}\left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \frac{C_w}{K} - \frac{\rho_e}{1+\tau_c}\frac{A}{K} - \delta, \tag{28}$$

$$\begin{aligned}\frac{\dot{A}}{A} &= \alpha\,(1-\tau_r)\,\frac{\psi-1}{\psi}\Lambda\left(\frac{A}{K}\right)^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}\left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} + (1-\tau_p)\,\frac{\alpha}{\psi}\Lambda\left(\frac{A}{K}\right)^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}-1}\left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} \\ &\quad -\rho_e - (1-\tau_r)\,\delta, \end{aligned} \tag{29}$$

$$\frac{\dot{C}_w}{C_w} = \alpha\,(1-\tau_r)\,\frac{\psi-1}{\psi}\Lambda\left(\frac{A}{K}\right)^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}\left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \rho_w - (1-\tau_r)\,\delta. \tag{30}$$

Notice that since Λ involves τ_c, and τ_w, the rates of consumption tax and the labor tax directly affect dynamic motions of K, A and C_w. On the other hand, the income share of government consumption, η, and the the rate of tax on profits, τ_p respectively have directly effects on dynamics of K and A, while the rate of capital tax directly affects behaviors of A and C_w

To simplify the dynamic system, let us denote

$$A/K = m, \quad C_w/K = s.$$

Then (28), (29) and (30) can be summarized by the following pair of differential equations:

$$\begin{aligned}\frac{\dot{m}}{m} &= \left[\frac{\alpha}{\psi}\,(\psi-1)\,(1-\tau_r) - (1-\eta)\right]\Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}} s^{-\frac{1-\alpha}{\alpha+\gamma}} + (1-\tau_p)\frac{\alpha}{\psi}\Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}-1} s^{-\frac{1-\alpha}{\alpha+\gamma}} \\ &\quad + s + \frac{\rho_e}{1+\tau_c}m + \tau_r\delta - \rho_e, \end{aligned} \tag{31}$$

$$\frac{\dot{s}}{s} = \left[\frac{\alpha}{\psi}\left(\psi - 1\right)\left(1 - \tau_r\right) - \left(1 - \eta\right)\right] \Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}} s^{-\frac{1-\alpha}{\alpha+\gamma}} + s + \frac{\rho_e}{1+\tau_c} m + \tau_r \delta - \rho_w. \tag{32}$$

Differential equations (31) and (32) constitute a complete dynamic system that describes motions of m and s.

3.3 Balanced-Growth Characterization

Existence of the Balanced-Growth Path

When the economy is in the balanced growth equilibrium, $m \,(= A/K)$ and $s \,(= C_w/K)$ stay constant over time. This means that K, A, Y, C_w, C_e and w change at a common rate, and r and N do not change. The steady state conditions are characterized by the following:

$$\begin{aligned} &\left[\frac{\alpha}{\psi}\left(\psi - 1\right)\left(1 - \tau_r\right) - \left(1 - \eta\right)\right] \Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}} s^{-\frac{1-\alpha}{\alpha+\gamma}} + (1-\tau_p)\frac{\alpha}{\psi} \Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}-1} s^{-\frac{1-\alpha}{\alpha+\gamma}} \\ &\qquad + s + \frac{\rho_e}{1+\tau_c} m + \tau_r \delta - \rho_e = 0 \end{aligned} \tag{33}$$

$$\left[\frac{\alpha}{\psi}\left(\psi - 1\right)\left(1 - \tau_r\right) - \left(1 - \eta\right)\right] \Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}} s^{-\frac{1-\alpha}{\alpha+\gamma}} + s + \frac{\rho_e}{1+\tau_c} m + \tau_r \delta - \rho_w = 0. \tag{34}$$

These conditions yield:

$$\left(1 - \tau_p\right) \frac{\alpha}{\psi} \Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}-1} s^{-\frac{1-\alpha}{\alpha+\gamma}} = \rho_e - \rho_w, \tag{35}$$

Using (34) and (35), we obtain

$$\begin{aligned} s &= \rho_w - \tau_r \delta + \left\{ \left[\frac{\psi}{\alpha}(1-\eta) - \left(\psi - 1\right)\left(1 - \tau_r\right)\right] \frac{\rho_e - \rho_w}{1 - \tau_p} - \frac{\rho_e}{1+\tau_c} \right\} m \\ &\equiv G\left(m\right). \end{aligned} \tag{36}$$

Equation (35) is rewritten as

$$s = \left[\frac{\alpha\left(1 - \tau_p\right)\Lambda}{\psi(\rho_e - \rho_w)}\right]^{\frac{\alpha+\gamma}{1-\alpha}} m^{-\frac{\alpha(\psi-1)+\gamma(\psi-\alpha)}{\psi(1-\alpha)}} \equiv H\left(m\right). \tag{37}$$

Consequently, equations (36) and (37) determine the steady-state levels of m and s.

Since Y, K, A, w and C_w grow at a common rate on the balanced-growth path, we focus on the balanced growth rate of C_w. From (30) and (37), we find that the balanced growth rate is expressed as

$$g = \frac{\dot{C}_w}{C_w} = \left[(\psi - 1)(1 - \tau_r) \frac{\rho_e - \rho_w}{1 - \tau_p} \right] m - (1 - \tau_r)\,\delta - \rho_w. \tag{38}$$

Thus the balanced-growth rate increases with the steady state level of m.

As for the existence of the balanced growth equilibrium, the above discussion leads to the following proposition:

Proposition 1 *Assume that*

$$\rho_e > \rho_w \text{ and } \left[\frac{\psi}{\alpha}(1 - \eta) - (\psi - 1)(1 - \tau_r) \right] \frac{\rho_e - \rho_w}{1 - \tau_p} > \frac{\rho_e}{1 + \tau_c}. \tag{39}$$

Then there is a unique and feasible balanced-growth equilibrium.

Proof. Under the above restrictions on parameter values, the graph of $H(m)$ is a monotonically decreasing function of m, whereas $G(m)$ monotonically increases with m. In addition, $G(0) = \rho_w > 0$ and $\lim_{m \to 0} H(m) = +\infty$. Thus there is a unique level of $m^* \in [0, 1]$ that establishes $G(m^*) = H(m^*)$, showing that there is a unique feasible balanced-growth path: see Figure 1. ■

Although the parameter restrictions given by the last condition in Proposition 1 are rather complex, we will show that numerical examples in which conditions for the existence of balanced growth path are satisfied under plausible specifications of parameter values.

Stability

As discussed in the next section, the representative agent version of the AK growth model with variable labor supply does not involve transition dynamics, so that the economy always stays on the balanced-growth path. By contrast, in our AK growth model with two types of agents, the economy has transition process if its initial state is out of the balanced growth equilibrium. Our dynamic system consists of one jump variable, $s\,(= C_w/K)$ and one non-jump variable, $m\,(= A/K)$. Therefore, if the balanced-growth equilibrium exhibits a saddle-point property, the equilibrium path is at least locally determinate and stable. Inspecting the approximated dynamic system linealized at the balanced-growth path, we find that under our restriction on parameter values given in Proposition 1, the balanced growth equilibrium satisfies saddle point stability.

Proposition 2 *If the conditions in* (39) *are satisfied, the balanced-growth equilibrium is locally determinate and stable.*

Proof. See Appendix. ■

Since the stable saddle path is one dimensional, when the iniitial value of m is historically given, m and s monotonically converge to their long-run equilibrium levels.

4 Long-Run Effects of Fiscal Policy

In this section we explore the long-run impacts of fiscal policy. Before discussing the policy effects in our model, it is useful to examine the growth effect of fiscal actions in the presence of homogeneous firms, which clarifies the role of firm heterogeneity in our argument.

4.1 The Representative Agent Economy

First, consider the representative agent economy where neither finacial friction nor firm heterogeneity exsists. There is a continuum of identical households with a unit mass. It is assumed that the households directly own firms that have identical production technology. In this standard formulation, the aggregate social technology that involves external effects is given by $Y = QKN^{1-\alpha}$ and the competitive factor prices are determined by $r = \alpha Q N^{1-\alpha} - \delta$ and $w = (1-\alpha) QKN^{-\alpha}$. The labor market equilibrium condition (25) is replaced with

$$CN^{\gamma} = \frac{1-\tau_w}{1+\tau_c} (1-\alpha) QKN^{-\alpha},$$

where C denotes the aggregate consumption. This condition gives the equilibrium level of hours worked as follows:

$$N = \left[(1-\alpha) Q \frac{1-\tau_w}{1+\tau_c} \right]^{\frac{1}{\alpha+\gamma}} \left(\frac{C}{K} \right)^{-\frac{1}{\alpha+\gamma}}. \tag{40}$$

Hence, the equilibrium levels of output and the rate of return to capital are repectively given by

$$Y = Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} K,$$

$$r = \alpha Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \delta.$$

We still assume that the government's budget is balanced in each moment by adjusting a lump sum tax. Thus the market equilibrium condition for the final goods is $\dot{K} = (1-\eta) Y - C - \delta K$. Also, the optimal consumption of the representative household follows $\dot{C} = C\left[(1-\tau_r) r - \rho - \delta\right]$. Consequently, the dynamic equations of aggregate capital and consumption respectively given by the following:

$$\frac{\dot{K}}{K} = (1-\eta) Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \frac{C}{K} - \delta, \tag{41}$$

$$\frac{\dot{C}}{C} = \alpha (1-\tau_r) Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \rho - (1-\tau_r)\,\delta. \tag{42}$$

In the above, ρ denotes the time discount rate of the representative household. Figure 2 depicts the graphs of (41) and (42). Here, we assume that $1 > \eta + \alpha (1-\tau_r)$. This condition corresponds to $1 > \eta + (\alpha/\psi)(\psi - 1)(1-\tau_r)$ assumed in Proposition 1. Figure 2 shows that there is a unique level of C/K that attains the balanced growth of C and K. The figure also displays that the dynamic behavior of C/K is globally unstable, which means that the economy always stays on the balanced-growth path.

In the balanced growth equilibrium, C and K grow at a common rate. From (41) and (42) the balaced growth condition is:

$$\left[1-\eta-\alpha(1-\tau_r)\right] \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C}{K}\right)^{*-\frac{1-\alpha}{\alpha+\gamma}} = \left(\frac{C}{K}\right)^{*} + \tau_r\delta - \rho, \tag{43}$$

where $(C/K)^*$ is the steady state value of C/K. From (40) the above equation is also written as

$$\left[1-\eta-\alpha(1-\tau_r)\right] N^{*1-\alpha} = \left(\frac{C}{K}\right)^{*} + \tau_r\delta - \rho, \tag{44}$$

and the balanced growth rate is expressed by

$$g = \frac{\dot{C}}{C} = \alpha \left(1 - \tau_r\right) N^{*1-\alpha} - \rho - \left(1 - \tau_r\right) \delta, \tag{45}$$

where N^* denotes the steady-state level of hours worked. Inspecting (40) , (43) , (44) and Figure 2, it is easy to see that the comparative statics results on the balanced-growth path can be summarized as Table 1.

	$\tau_r \uparrow$	$\tau_w \uparrow$	$\tau_c \uparrow$	$\eta \uparrow$
$(C/K)^*$	$+$	$+$	$+$	$-$
N^*	$-$	$-$	$-$	$+$
g	$-$	$-$	$-$	$+$

Table 1: Long-Run Policy Impacts in the Representative Agent Economy

As the table shows, rises in τ_w and τ_c depress the balanced growth rate, g. It is easy to obtain intuitive implications of these results. For example, suppose that the rate of wage income tax, τ_w, rises Equation (40) states that a higher τ_w enhances the labor wedge,which depresses the steady state level of hours worked, N^*. Since (45) shows that the balanced growth rate increases with N^*, a rise in τ_w lowers the balanced growth rate. The same intuition applies to the growth effect of an increase in the rate of consumption tax, τ_c

A rise in the rate of tax on capital income, τ_r, yields a negative effect on the after-tax rate of return, $\left(1 - \tau_r\right) r$, which lowers the households' saving and the rate of capital expansion is reduced. As a result, C/K increases, so that from (40) N^* decreases. On the other hand, a higher τ_r reduces the 'after-tax' rate of capital depreciation, $\left(1 - \tau_r\right) \delta$, . which has a positive effet on capital accumulation. However, as long as the balanced growth rate is positive, the negative effect on the after-tax rate of return dominates the positive effect on $\left(1 - \tau_r\right) \delta$, so that a rise in τ_r reduces g.[16]

As to the growth effect of a change in η, note that a rise in the income share of government consumption crowds out private investment and consumption. In addition, a higher government consumption raises a lump sum tax on the households' income, which yields a further

[16] To have a positive balanced growth rate, it should hold that $g = \dot{C}/C = \left(1 - \tau_r\right)\left(r - \delta\right) - \rho > 0$ so that $r > \delta$. Therefore, under a give level of r, $dg/d\tau_r = -r + \delta < 0$, implying that $\dot{C}/C$ line shifts downward in Figure 2.

reduction in consumption. Accordingly, the steady state level of C/K is reduced so that from (40) N^* increases, which raises the rate of return to capital and thus the balanced-growth rate rises. It is to be pointed out that the positive effect of a rise in η on g depends on our assumption that a higher government consumption is financed by adjusting lump sum tax levied on workers. If a rise in η is associated with increases in tax rates, then a higher η may lowers g. We discuss this point again in Section 5.3.[17]

4.2 Homogeneous Firms with Financial Frictions

Next, consider the second special case where the production technology owned by each entrepreneur is homogeneous, but financial constraints remain. This means that the shape parameter is given by $\psi = \infty$ and the aggregate capital and aggregate asset held by the entrepreneurs satisfy $K = \theta A$. Since firms are homogeneous, there is no entry barrier to production: when production takes place, all the firms produce so that there are no excess profits and the competitive rate of return to capital equals the marginal productivity of aggregate capital. Therefore, the asset held by the entrepreneurs follows

$$\frac{\dot{A}}{A} = (1 - \tau_r)\left(\alpha \frac{Y}{K} - \delta\right) - \rho_e.$$

Observe that consumption of workers changes according to

$$\frac{\dot{C}_w}{C_w} = (1 - \tau_r)\left(\alpha \frac{Y}{K} - \delta\right) - \rho_w.$$

These two equations mean that the balanced growth equilibrium exists only if $\rho_e = \rho_w = \rho$. Given this condition, C_w/K stays constant over time.

The equilibrium level of hours worked given by (26) is replaced with

$$N = \left[(1 - \alpha) Q \frac{1 - \tau_w}{1 + \tau_c}\right]^{\frac{1}{\alpha+\gamma}} \left(\frac{C}{\theta A}\right)^{-\frac{1}{\alpha+\gamma}}.$$

[17] See Turnovsky (2000) for a detailed policy experiments in the Ak growth model with variable labor supply.

Thus the aggregate production function is written as

$$Y = Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C_w}{\theta A}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} K.$$

To determine the steady state level of C_w/A, we use the capital accumulation equation such that

$$\frac{\dot{K}}{K} = (1-\tau_r) Q^{\frac{1+\gamma}{\alpha+\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1-\alpha}{\alpha+\gamma}} \left(\frac{C_w}{K}\right)^{-\frac{1-\alpha}{\alpha+\gamma}} - \frac{C_w}{K} - \frac{C_e}{K} + \tau_r \delta - \delta, \qquad (46)$$

where $\rho_e = \rho_w$ and $C_e/K = \rho_e/\theta\,(1+\tau_c)$. Equations (42) and (46) constitute a complete dynamic system with respect to K and C_w. Consequently, the dynamic system is exactly the same as that of the representative agent economy except that the consumption function of the entrepreneurs is $C_e = \rho_e A/\theta\,(1+\tau_c)$ rather than $C_e = \rho_e A/\,(1+\tau_c)$.

The above discussion reveals that the presence of firm heterogeneity plays a crucial role in our model. This is because the firm heterogeneity yields an endogenously determined cutoff level of production efficiency, which affects the TFP of the aggregate production function. In the absence of firm heterogeneity, TFP is fixed and the financial constraints do not yield essential effects (except for the effect on the entrepreneurs' consumption function).

4.3 Heterogeneous Firms with Financial Frictions

Our model involves two parameters that do not appear in the representative agent model: the shape parameter of the Pareto distribution, ψ, and the tightness of the financial constraint, θ. To study the fiscal impacts in our model, it is useful to inspect the relation between the long-term growth and these two parameters The former represents the degree of heterogeneity of firms (a lower ψ means a higher degree of firm heterogeneity), the latter reflects the degree of financial development. First, it is easy to see that if the degree of financial constraint decreases (θ has a higher value), then the graph of $H(m)$ shifts upward, which brings abut increases in both m and s. Thus from (38) the long-run growth rate increases. As was expected, a reduction in the degree of financial friction realizes a more efficient resource allocation, which raises the long-term growth performance of the economy.

On the other hand, a decreases in the degree of heterogeneity of firm, i.e. a rise in ψ, yields a downward shifts of the graphs of both $G(m)$ and $H(m)$. Thus its effect on the steady state level of m is ambiguous. In view of (38), an increase in ψ gives a direct positive effect on the balanced growth rate, but its total effect depends on the change in m as well. To sum up, we find:

Proposition 3 *Under the conditions in* (39), *an economy with a lower degree of financial frictions attains a higher rate of balanced growth, while the growth effect of a decrease in the degree of heterogeneity of firms is qualitatively ambiguous.*

Capital Income Tax

Keeping the above facts in mind, let us explore the growth effect of each fiscal policy. First, suppose that the rate of capital income tax, τ_r, rises permanently. Figure 1 demonstrates that a higher τ_r shifts the graphs of $G(m)$ upward. As a consequence, the steady-state level of m decreases and that of s increases, which leads to a reduction in the hours worked, N: see equation (40). Remember that from (18) the balanced-growth rate of the workers' consumption is also expressed as

$$g = \alpha (1 - \tau_r) Q \left(\frac{\psi}{\psi - 1} z^* \right)^{\alpha} N^{1-\alpha} - (1 - \tau_r) \delta - \rho_w.$$

Since a reduction in m lowers the cutoff, z^*, a rise in τ_r yields three negative impacts on long-run growth, that is, the direct effect on the after tax rate return and the indirect effects through reductions in TFP and the aggregate employment. It is to be noted that, as pointed out in Section 4.1, an increase in τ_r lowers $(1 - \tau_r)\delta$, but this effect does not dominate the negative impacts mentioned above, as long as the balanced growth rate takes a positive value.

Labor Income Tax and Consumption Tax

As for the quantitative effects of taxation on wage income and on consumption expenditure are similar to those observed in the representative agent economy. As well as in the representative agent economy, both taxes enhance the labor wedge in the workers' optimal choice between consumption and labor. An increase in τ_w gives rise to a downward shift of $H(m)$ in Figure 1. The steady-state levels of m and s decrease simultaneously. Although the effect on the steady-state value of N is ambiguous, equation (38) states that the balanced-growth rate is reduced by a decrease in m. As to the effects on the steady-state values of m, s and N, an increase in the rate of consumption tax has the same outcomes as that of a rise in the labor income tax. A difference from the representative agent model is that a change in τ_c also affects the entrepreneurs' aggregate consumption, $C_e = \rho_e A/(1+\tau_c)$.

Profit Tax

Figure 1 shows that a rise in π_p decreases the steady-state value of m. However, (38) shows that a higher π_p has a direct positive effect on growth under our assumption that $\rho_e > \rho_w$. As a result of this positive effect, the impact of a change in τ_p on the balanced-growth rate is qualitatively undetermined.[18]

Government Consumption

A rise in η makes a downward shift of the graph of $G(m)$. Hence, the steady state level of s decreases, whereas that of m increases. This shows that from (38) the balanced-growth rate will increase. Such a positive effect of government consumption on long-run growth is the same as that in the representative agent economy. However, the source of its impact is different from the case of homogeneous firms. In our model a higher income share of government consumption crowds out private investment, which leads to a decline in the rate of capital accumulation. This increases the leverage ratio, A/K, so that the cutoff level of efficiency, z^*, rises. As a result, the average productivity of active firms rise so that the productivity of the aggregate technology increases as well. Hence, we obtain a higher rate of balanced growth.

Our discussion so far is summarized in Table 2 and Proposition 4 listed below:

	$\tau_r \uparrow$	$\tau_w \uparrow$	$\tau_c \uparrow$	$\tau_p \uparrow$	$\eta \uparrow$
$m^* (= A/K)$:	$-$	$-$	$-$	$-$	$+$
$s^* = (= C_w/K)$:	$+$	$-$	$-$	?	$-$
N^* :	$-$	?	?	?	$+$
g :	$-$	$-$	$-$	?	$+$

Table 2: Long-Run Policy Impacts in the Heterogeneous-Agent Economy

[18] In the next section we demonstrate that a rise in π_p yields a significant negative effect on growth under plausible magnitudes of parameters.

Proposition 4 *Under the conditions in* (39) , *the balanced growth rate of the heterogeneous agent economy is negatively related to the rates of tax on capital income, labor incomes and consumption spending, whereas it is positively related to the income share of government consumption. The growth effect of a changes in the rate of profit tax is qualitatively undetermined..*

To sum up, except for the profit tax (which does not exist in the representative agent economy), the effects of fiscal policy on the long-run growth rate are similar to those in the representative agent economy at least in the qualitative sense. To evaluate quantitative differences in the long-run fiscal impacts between the two economies, we examine some numerical examples in the next section.

5 Numerical Analysis

5.1 The Representative Agent Economy

We first conduct numerical experiments in the case of representative agent model. The baseline setting of the key parameter values are as follows:

$$\alpha = 0.7, \quad \tau_r = \tau_w = 0.3, \quad \tau_{c:} = 0.1, \qquad \eta = 0.15, \quad \rho = 0.03, \; \gamma = 1, \qquad \delta = 0.05, \quad Q = 0.402$$

According to the standard specifications in the RBC literature, we set the magnitudes of income share of labor, α, the time discount rate, ρ, the elasticities of labor supply. $1/\gamma$ at at their conventional levels. The baseline rates of capital and wage income tax is assumed to be 0.3, the rate of consumption tax, $\tau_c = 0.1$ and the income share of government consumption, $\eta = 0.15$. Finally, we set the scale parameter $Q = 0.402$ in order to hold that the baseline growth rate under the policy parameters given above is 2% per year. The relation between each policy varaaible and the balanced growth rate is displayed in Table 3.

Table 3: Growth Effect of Fiscal Policy in the Representative Agent Economy

τ_r	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.048	0.043	0.037	0.032	0.038	0.023	0.020	0.017	0.011	0.007	0.004

Table 3-a

τ_w	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.029	0.028	0.027	0.026	0.024	0.022	0.020	0.019	0.017	0.015	0.012

Table 3-b

τ_c	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.0223	0.021	0.020	0.019	0.018	0.0177	0.01706	0.0165	0.0160	0.0156	0.0153

Table 3-c

η	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.012	0.016	0.018	0.020	0.023	0.027	0.030	0.035	0.041	0.046	0.06

Table 3-d

In Table3-a, keeping $\tau_{w,}$, τ_c and η at their baseline values, we change τ_r from 0 to 0.5.(When $\tau_r = 0.2$, the balanced growth rate, g, is 0.02.) This table reveals that the growth impact of a change in the rate of capital income tax is substantially large in the representative agent economy. The balanced-growth rate monotonically decreases as τ_r increases: it drops from about 4.8% for $\tau_r = 0$ to 0.004% for $\tau_r = 0.5$. The steady-state level of C/K is about 0.074 when $\tau_r = 0$, while it is 0.11 when $\tau_r = 0.5$ Remember that a higher C/K means a lower hours worked and a lower income growth. Remember that a change in τ_r has such an indirect effect on labor supply as well as the direct effect on the after-tax rate of return to capital. Since both effects are numerically substantial in our model, a higher τ_r gives rise to a relatively large decline in the long-run growth rate.

In Table 3-b and c, we conduct the same experiments as to τ_w,and τ_c. Those tables state that in contrast to the rate of tax on capital income, the growth effects of taxation on wage income and consumption spending are relatively small. In particular, when there is no consumption tax, the balanced growth rate is about 2.2% as opposed to 2% for $\tau_c = 0.1$. Even when the consumption tax rate is 50%, the economy still grows at the rate of 1.5%. In the case of wage income tax, the growth effect is more visible than that of the consumption tax. The balanced growth rate is about 2.9% for $\tau_w = 0$, while it is 1.2% for $\tau_w = 0.5$. In the cases of wage income tax and consumption tax, their growth effects stem from the indirect effect of a changes in C/K (so changes in hours worked, N) and from the change in labor wedge expressed by $(1 - \tau_w) / (1 + t_c)$. Since a change in C/K caused by a change in τ_w or in τ_c is sufficiently small, the main source of effect of a change in the labor wedge. An increase in τ_w linearly raises the labor wedge, while the marginal effect of a change in τ_c on the labor wedge diminishes as τ_c increases. As a result, the growth effect of consumption tax is the weakest among alternative tax policies.

As sbown by Table 3-d, the growth effect of the income share of government consumption, η, is relatively large. The balanced-growth rate raises from 1.2% for $\eta = 0$ to 6% for $\eta = 0.5$. As mentioned above, a rise in the government share of consumption crowds out private consumption, which lowers the consumption-capital ratio on the new balanced-growth path. This enhances the equilibrium level of hours worked and thus the long-term growth rate increases.

5.2 The Heterogeneous-Agent Economy

The baseline values of parameters in our model economy are as follows:

$$\begin{aligned} \alpha &= 0.7, \quad \rho_w = 0.01, \quad \rho_e = 1.0, \quad \delta = 0.03. \quad \gamma = 1, \ , \quad \psi = 1.6, \quad \theta = 2.5, \quad Q = 0.101, \\ \tau_r &= \tau_w = \pi_p = 0.3, \quad \tau_c = 0.1, \quad \eta = 0.15 \end{aligned}$$

The magnitudes of α and γ are the same as before. The baseline fiscal policy parameters are also the same as those in the representative agent economy. The magnitude of the shape parameter of distribution of production efficiency, z, follows Jaimovich and Rebelo (2014) who rely on the estimation of the Pareto coefficient of the US income distribution conducted by Diamond and Saez (2011). We set $\theta = 2.5$ and $Q = 0.101$. Here, both parameters play the role of scale parameters in quantitative evaluation of the model. In this example we assume that the entrepreneurs are much more impatient than the workers so that $\rho_e = 0.10$ and $\rho_w = 0.01$.[19] We also adjust the depreciation of capital to make the balanced growth rate 2% under the baseline levels of policy parameters. The rate of capital depreciation (3% per year) is obviously too small. However, if we set $\delta > 0.05$,the balanced growth rate is mostly negative in our examples. The results of numerical experiments are summarized in Tables 4.

Table 4: The Growth Effects of Fiscal Policy in the Heterogeneous Agent Economy

[19] If the divergence between ρ_e and ρ_w is small, the balanced-growth rate takes implausibly high value.

τ_r	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	$0,037$	0.34	0.032	0.028	0.025	0.022	0.20	0.017	0.014	0.012	0.009

Table 4-a.

τ_w	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.026	0.024	0.023	0.023	0.022	0.021	0.020	0.018	0.017	0.015	0.011

Table 4-b.

τ_c	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.025	0.022	0.020	0.019	0.017	0.015	0.014	0.013	0.012	0.011	0.010

Table 4-c.

η	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.014	0.015	0.017	0.020	0.022	0.024	0.025	0.0256	0.027	0.028	0.029

Table 4-d

τ_p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
g	0.112	0.084	0.063	0.048	0.035	0.026	0.02	0.0256	0.005	-0.014	-0.031

Table 4-e.

Table 4-a displays that a change in the rate of capital income tax yields a sizable negative effect on the long-run growth rate. However, its impact is smaller than that observed in the representative agent economy. The steady-state level of $m = 0.98$ for $\tau_r = 0$ and $m = 0.81$ for $\tau_r = 0.5$: a 50% increase in the rate of capital income tax lowers the relative asset holding of entrepreneurs from about 1.0 to 0.8. Despite the large increase in τ_r, such a decrease in m is relatively small so that the policy impact on the hours worked is also small.

Similarly, the growth effect of the income share of government consumption, η, is smaller than that in the representative agent economy: see Table 4d. As shown in Table 2, a higher η raises m and reduces s on the balanced growth path. Both effects have positive impacts on TFP as well as on the hours worked, N. However, their impacts may be relatively small, compared to the effects in the representative agent economy where only N^* is adjusted in response to a change in η.

In Section 4.1 we pointed out that the positive relation between η and g depends on our assumption that an increase in the government consumption is financed by a change in the lump sum tax on workers. Alternatively, suppose that a part of government consumption is financed by distortionary income taxation. For example, let us assume that the economy stays on the balanced growth path and policy variables take the baseline magnitudes used in our example. Then the government raises η and finances it by an increase in the wage income taxation to keep $\Delta G = \Delta \eta Y = \Delta \tau_w w N$. Since $wN = (1-\alpha) Y$, this assumption means that $\Delta \tau_r = \Delta \eta / (1-\alpha)$. If η rises from 0.15 to 0.2, then τ_w increases from 0.3 to 0.43. In our baseline example, these simultaneous changes in policy parameters decreases the balanced growth rate. By the same token, it is easy to see that if an increase in η raises τ_r or π_p, then a higher income share of government consumption yields a substantial decrease in the balanced growth rate.

As to the rate of tax on labor income, its quantitative negative effect on the long-run growth rate is smaller than that in the representative agent economy, but the differences are not significant. On the other hand, the growth effect of a change in the rate of consumption tax in our model is relatively larger than that in the representative agent model. Such a difference may stem from the fact that in our model the entrepreneurs' consumption expenditure is $C_e = A/(1+\tau_c)$, so that an increase in τ_c depresses entrepreneurs' consumption, which accelerates capital accumulation. This may lowers A/K, which enhances the negative impact of consumption tax on growth through a reduction of cutoff level z^*.

Finally, in our example, the long-term growth rate is quite sensitive to a change in the rate of tax on profit income. Table 4-e demonstrates that the balance growth rate changes from 11.3% to -3.5% as π_p increases from 0 to 0.5. In particular, the cutoff level z^* rapidly decreases as π_p rises. Since a higher τ_p reduces active entrepreneurs' after tax income, so that their asset accumulation is lowered. This yields a substantial decrease in the steady state level of $m(= A/K)$ and, hence, the cutoff z^* will decline substantially. As seen in (38), a rise in τ_p yields a direct positive effect on the growth. However, this positive effect is not large enough to cancel the negative effect on m, which leads to a large decrease in the long-run growth rate.

5.3 Transition Dynamics

Unlike the representative agent model with an AK technology, our model involves transition dynamics. Based on the baseline magnitudes of policy parameters ($\tau_r = \tau_w = \tau_p = 0.3$, $\tau_c = 0.1$ and $\eta = 0.15$), we find that the phase diagram of dynamic equations (31) and (32) near the steady state can be depicted by Figure 3-a. Both $\dot{m} = 0$ and $\dot{s} = 0$ loci have negative slopes and $\dot{s} = 0$ locus is steeper than $\dot{m} = 0$ locus. As shown in the figure, the stable saddle path converging to the balanced growth equilibrium is negatively sloped as well.

As an example of transitional impacts of fiscal policy, suppose that the economy initially stays on the balanced growth path and that there is an unancipated permanent reduction in the rate of tax on capital income. A fall in τ_r shifts up both $\dot{m} = 0$ and $\dot{s} = 0$ loci in Figure 3-a and, hence, the stable saddle path also shifts upward. As shown in Figure 3-b, after the policy change the economy jumps from the initial position E_0 up to point E_1 on the new saddle path. Namely, there is an instantaneous rise in C_w and then the economy converges to the new steady state E_2. During the transition, s continues to decrease, while m continues to increase. Thus it holds that $\dot{A}/A > \dot{K}/K > \dot{C}_w/C_w$ on the converging process. Furthermore, we see that during transition, the average productivity $\psi/\left(\psi - 1\right) z^* = \psi/\left(\psi - 1\right) \left(m\right)^{\theta/\psi}$ and the hours worked, N, continue rising. The transition impacts of changes in other fiscal actions can be examined in the similar manner.

Next, let us consider the speed of convergence of our economy. We evaluate the coefficient matrix of the linearized dynamic system shown in the Appendix. Using the baseline parameter values, we see that the absolute value of the stable root of the coefficient matrix is about 0.042. This means that the time length to adjust 90% of transition towards the new steady state takes about 23 years and 50% adjustment takes about 11 years. Table 4-a shows that if τ_t falls from 0.3 to 0.2, then the balanced growth rate increases from 2% per year to 2.5% per year.[20] Notice that in the representative agent counterpart of our model, an unanticipated permanent policy change yields an instantaneous jump of the economy on the new balanced growth path. In contrast, in our model a change in the rate of capital income tax yields not only an impact on the long-run growth rate but also on the dynamic response of the aggregate economy.

[20] When endogenous growth models involve transition dynamics, the convergence speed is generally faster than the neoclassical (exogenous) growth models. The adjustment speed of our example is close to that obtained in endogenous growth models with physical and human capital: see Ortigueira and Santos (1997).

6 Concluding Remarks

This paper constructs a tractable model of endogenous growth with financial frictions and firm heterogeneity. We introduce various tax policies and government consumption into the base model and examine the long-run effect of fiscal actions. We find that from the qualitative perspective, most of the policy effects are essentially the same as those established in the corresponding representative agent model without financial frictions. However, our numerical experiments demonstrate that quantitative impacts of fiscal policy may be substantially different from the effects observed in the representative agent model. For example, the growth effects of capital income tax and government consumption are smaller in our heterogeneous agent model than in the representative agent counterpart. In contrast, the consumption tax may yield a larger growth effect in our model than in the representative agent model. Furthermore, the rate of profit income tax, which is not present in the representative agent economy, may have a large growth effect in our setting. Of course, numerical outcomes are quite sensitive to the model structure and as well as to the specifications of parameter values. Therefore, we should not put too much emphasis on our specific examples. Nevertheless, our discussion reveals that the long-run effects of fiscal policy may critically depend upon the structure of the model economy. This suggests that we should carefully re-examine the roles of fiscal policy discussed in the conventional endogenous growth models where agents are homogeneous and the financial market are free from frictions.

In this paper we have mainly focused on the effect of each policy change on the balanced-growth rate. Although we have briefly discussed transition dynamics, the short-run impact of fiscal policy should be examined in detail. Also, the base model can be extended by adding public investment, government debt and income distribution policy. As for the formulation of financial constraint, we use the standard collateral constraint on investment. It is interesting to explore the fiscal impacts under alternative forms of financial constraints.[21] Finally, we have assumed that idiosyncratic shocks, z are *iid* over time as well as across agents. This assumption greatly simplifies the analytical discussion, but using more general formulations

[21] Chen and Mino (2014) use the formulation of financial frictions as that assumed in this paper and discuss the working of neoclassical (exogenous) model. As for the roles of alternative forms of financial constraints, see, for example, Benhabib and Wang (2013), Jermann and Quadrini (2012) and Miao and Wang (2012). Quadrini (2011) presents a comprehensive survey over the alternative formulations of financial frictions in macroeconomic models.

used in the firm dynamics literature may deepen our understanding.[22] Those topics deserves further study.

[22] For more general treatment of growth models with heterogeneous firms, see Luttmer (2010) and Nirei and Aoki (2014).

Appendix

Proof of Proposition 2

To inspect stability of the dynamics system, let us express (31) and (32) in the following manner:

$$\frac{\dot{m}}{m}=\left[1+\frac{(\alpha/\psi)(1-\tau_p)}{(\alpha/\psi)(\psi-1)(1-\tau_r)-(1-\eta)}\frac{1}{m}\right]\Delta(m,s)+s+\rho_e m-\rho_e,$$

$$\frac{\dot{s}}{s}=\Delta(m,s)+s+\rho_e m-\rho_w,$$

where

$$\Delta(m,s)=\left[\frac{\alpha}{\psi}(\psi-1)(1-\tau_r)-(1-\eta)\right]\Lambda m^{\frac{\alpha(1+\gamma)}{\psi(\alpha+\gamma)}}s^{-\frac{1-\alpha}{\alpha+\gamma}},$$

which has a negative value due to our assumption made in Proposition 1. Given our assumptions in Proposition 1, we see that $\Delta_m<0,\ \Delta_s>0$ and $\Delta<0$. The coefficient matrix of the above system linearized at the steady state is

$$J=\begin{bmatrix} m^* & 0\\ 0 & s^*\end{bmatrix}\begin{bmatrix}\left(1+\frac{\Phi}{m^*}\right)\Delta_m-\Phi\frac{\Delta}{m^{*2}}+\rho_e, & \left(1+\frac{\Phi}{m^*}\right)\Delta_s+1\\ \Delta_m+\rho_e, & \Delta_s+1\end{bmatrix}.$$

where

$$\Phi=\frac{(\alpha/\psi)(1-\tau_p)}{(\alpha/\psi)(\psi-1)(1-\tau_r)-(1-\eta)}<0.$$

Then we derive:

$$\begin{aligned}\det J &= m^*s^*\det\begin{bmatrix}-\Phi\frac{\Delta}{m^{*2}}-\frac{\Phi}{m^*}\rho_e & -\frac{\Phi}{m^*}\\ \Delta_m+\rho_e & \Delta_s+1\end{bmatrix}\\ &= -\frac{s^*}{\Phi}\left[\frac{\Delta}{m^*}(\Delta_s+1)+\Delta_m-\rho_e\Delta_s\right].\end{aligned}$$

Since $\Delta<0,\ \Delta_m<0,\ \Delta_s>0$ and $\Phi<0$, the sign of the determinant of J is negative, implying that the steady state of our dynamic system holds a saddle point property.

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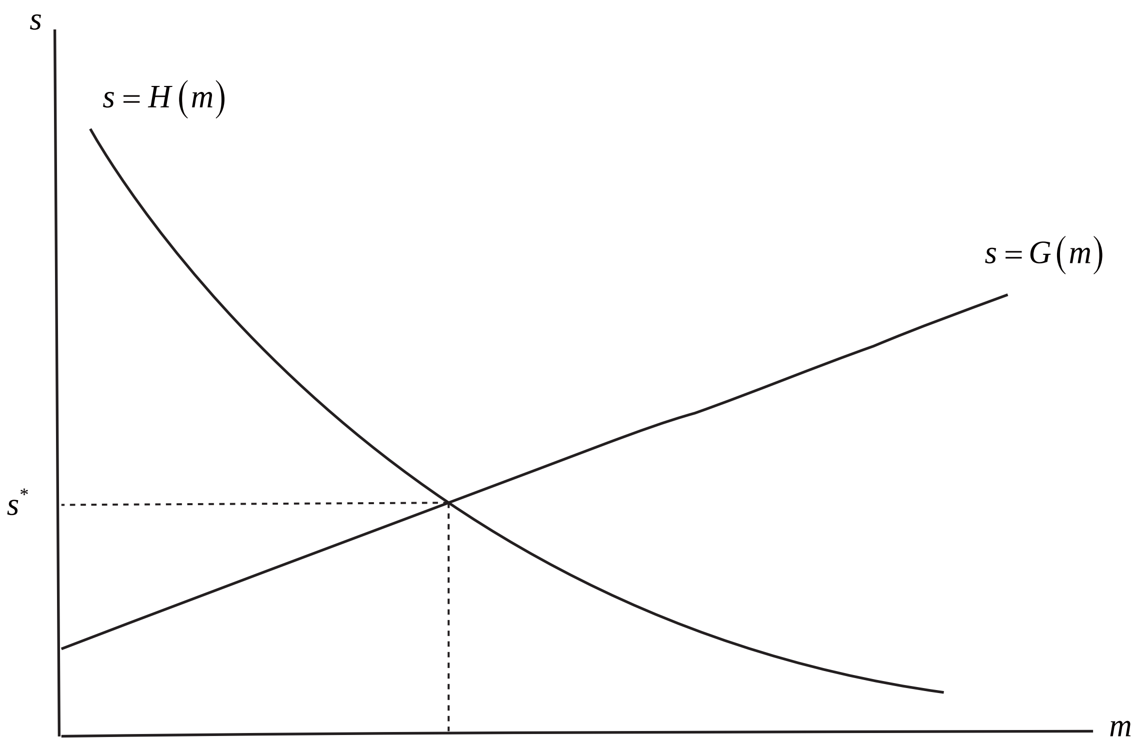

Figure 1: Existence of the Balanced-Growth Equilibrium

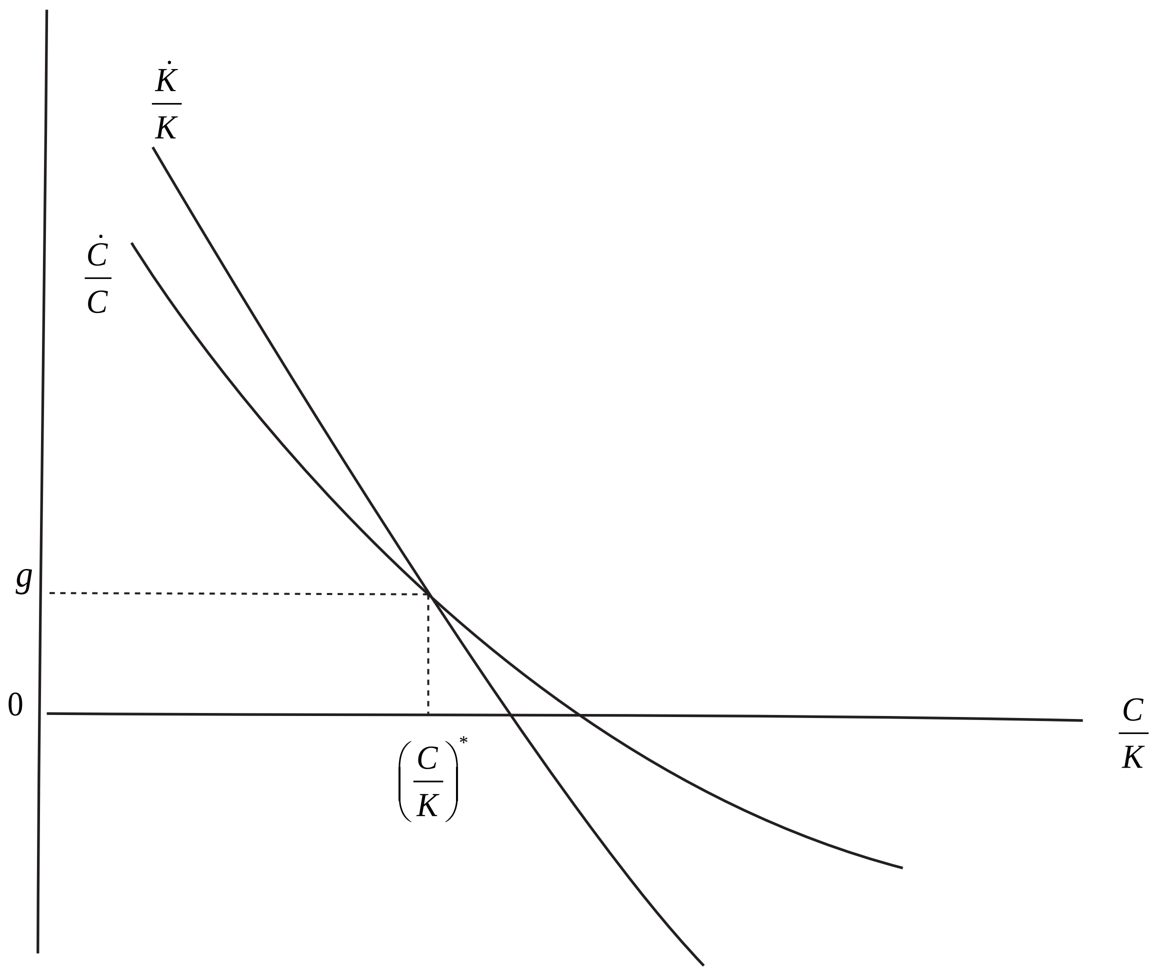

Figure 2: The Balanced-Growth Equilibrium of the Representative-Agent Economy

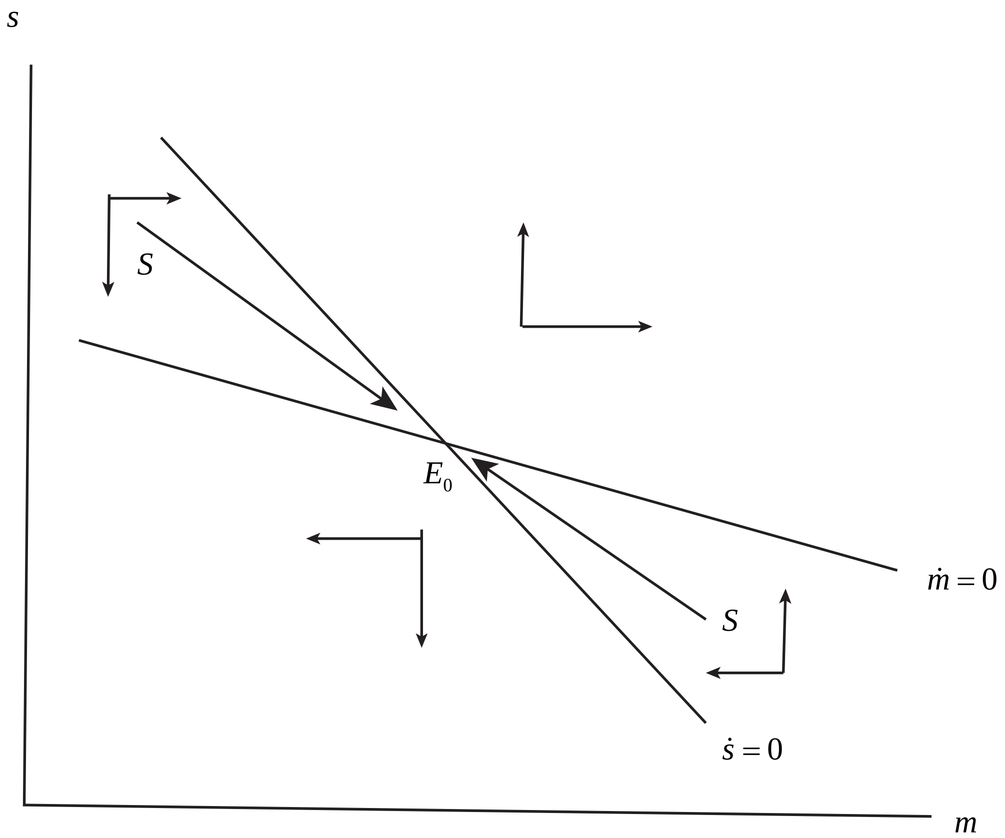

Figure 3-a: Stability of the Balanced Growth Equilibrium

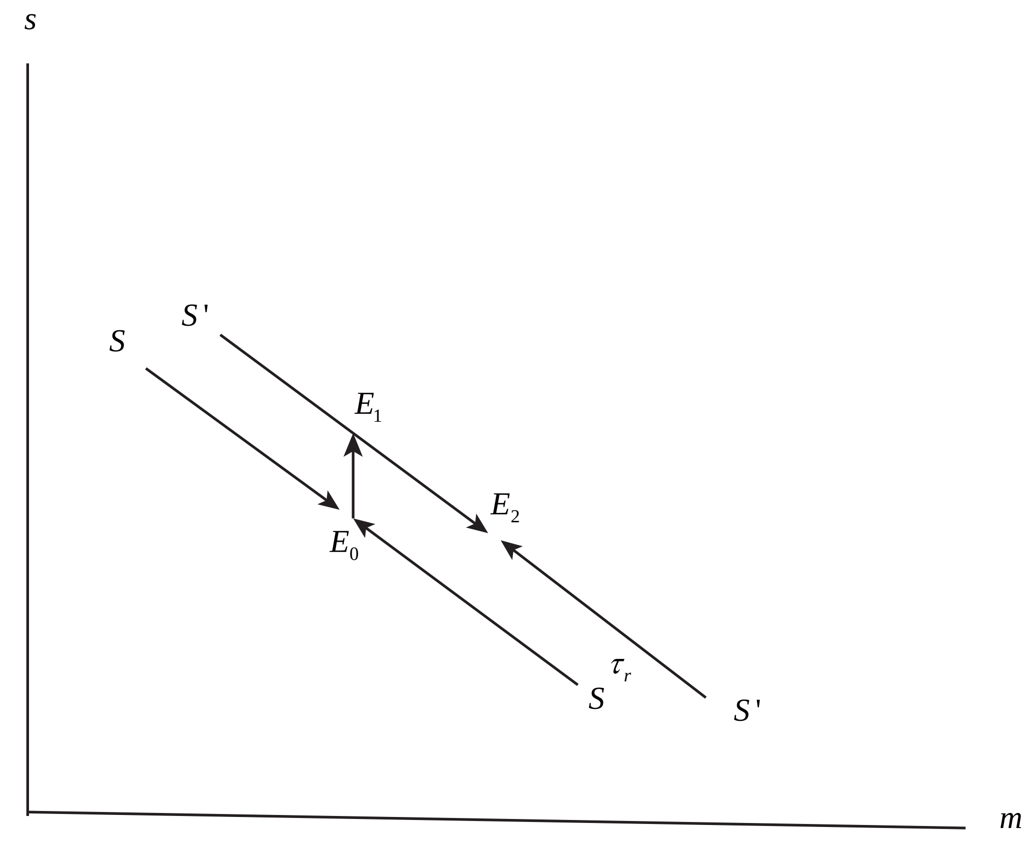

Figure 3-b: The Transition Path after a Rise in τ_r