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Abstract

This paper constructs a simple model of endogenous growth with financial frictions and firm heterogeneity. In the presence of financial constraints and heterogeneity in production efficiency of firms, the firms whose efficiency exceeds the cutoff level produce and the entrepreneurs who own those firms become borrowers. We show that even if production technology of each firm has an Ak property, the aggregate economy has transition dynamics and that the balanced growth rate depends on the aggregate distribution of wealth between rentiers and entrepreneurs.

keywordes: financial frictions, firm heterogeneity, endogenous growth, wealth distribution

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1 Introduction

This paper constructs a simple model of endogenous growth that involves financial frictions and firm heterogeneity. Our model economy consists of two classes of agents: rentiers and entrepreneurs. Rentiers consume final goods and accumulate riskless bonds. Each entrepreneur owns a firm. Productivity of each firm is assumed to be heterogeneous. Due to the presence of financial constraints, there is an endogenously determined cutoff level of production efficiency. The entrepreneurs who draw efficiency levels that are higher than the cutoff produce and become borrowers. On the other hand, the entrepreneurs who draw efficiency levels that are less than the cutoff give up production and become lenders. In this setting, it is shown that even though production technology of each firm is a simple Ak type, the aggregate economy exhibits transition dynamics. It is also revealed that the balanced-growth rate depends on the distribution of aggregate wealth between rentiers and entrepreneurs.

There are several ways of formulating a dynamic macroeconomic model with financial frictions.[1] Our formulation is closed to that used by Moll (2014) and Itskhoki and Moll (2014). While these authors focus on closed as well small-open economies with exogenous growth, this paper explores a closed economy model in which endogenous growth is sustained. Our setting is also similar to the model studied by Liu and Wang (2013) who discuss equilibrium indeterminacy in an exogenous growth model with financial constraints and firm heterogeneity. A merit of our model is that it is simple enough to elucidate the effects of financial frictions on long-run growth in a clear, tractable manner.

2 Model

In our economy there is a continuum of identical rentiers with a unit mass. They do not participate in production activities. The representative rentier maximizes a discounted sum of utilities

$$U^r = \int_0^\infty e^{-\rho_r t} \log C_r dt, \qquad \rho_r > 0$$

subject to the flow budget constraint

$$\dot{B} = rB - C_r, \tag{1}$$

[1] See Quandrari (2011) for a comprehensive survey over alternative modelling of financial frictions in macroeconomics.

as well as to the no-Ponzi game scheme such that $\lim_{t\to\infty} \exp\left(-\int_0^t r_t dt\right) B_t \geq 0$. Here, C_r and B repetitively denote consumption and bond holdings of retires, and r is the real interest rate. The assumption of log utility yields the consumption function such that

$$C_r = \rho_r B. \tag{2}$$

There also exists a continuum of entrepreneurs with a unit measure. Each entrepreneur owns a firm. Firms produce homogeneous final goods using capital alone. The production function of each firm is

$$y_i = Qz_i k_i, \quad i \in [0,1], \tag{3}$$

where, y_i and k_i repetitively denote output and capital. Here, z_i represents the production efficiency of the firm owned by type i entrepreneur. In each moment, each entrepreneur draws an efficiency level of capital from a Pareto distribution whose cumulative distribution is given by

$$F(z) = 1 - z^{-\psi}, \quad \psi > 1. \tag{4}$$

In the above, a lower level of the shape parameter ψ means a higher degree of heterogeneity in production technology. Following Itskhoki and Moll (2014) and Liu and Wang (2014), we assume that z is iid over time as well as across agents. Therefore, the share of agents who draw a particular value of z is deterministic.

Each entrepreneur maximizes an expected sum of utilities

$$U_i^e = E_0 \int_0^\infty e^{-\rho_e t} \log c_i^e dt, \quad \rho_e > 0$$

subject to the flow budget constraint

$$\dot{d}_i = rd_i + c_i^e + \dot{k}_i + \delta k_i - y_i,$$

where d_i is debt, c_i^e is consumption of an entrepreneur and $\delta \in [0,1)$ denotes the depreciation rate of capital. In addition to the budget constraint, each entrepreneur faces with a debt constraint such that

$$d_i \leq \lambda k_i, \quad 0 < \lambda < 1.$$

Namely, the debt limit is proportional to the capital employed by the entrepreneur. We assume that the commitment of borrowers are limited, so that the parameter λ is strictly less than one.

Letting $a_i = k_i - d$ be the net worth held by type i entrepreneur, we may rewrite the flow budget and debt constraints as follows:

$$\dot{a}_i = y_i - (r + \delta) k_i - c_i^e, \tag{5}$$

$$k_i \leq \theta a_i, \quad \theta = \frac{1}{1-\lambda} > 1. \tag{6}$$

Hence, if $\theta = +\infty$ $(\lambda = 1)$, then the financial market is perfect, while no borrowing is allowed if $\theta = 1$ $(\lambda = 0)$.

We first characterize the entrepreneur's instantaneous production decision. Each entrepreneur maximizes the excess profit, $\pi_i = y_i - (r + \delta) k_i$, under the constraints of (3) and (6). The first-order conditions for an optimum are:

$$Qz_i = r + \delta + \mu_i, \tag{7}$$

$$(\theta a_i - k_i) \mu_i = 0, \quad \mu_i \geq 0, \qquad \theta a_i - k_i \geq 0. \tag{8}$$

where μ_i is the Lagrangean multiplier associated with the debt constraint. When the financial constraint does not bind, then $\mu_i = 0$. Thus μ_i represents the investment wedge that diverges the the marginal product of capital from the real interest rate. We assume that entrepreneurs produce as long as excess profits are non negative. Since it holds that $\pi_i = \mu_i k_i$, entrepreneurs produce if and only if μ_i has a nonnegative value. As a result, the cutoff level of z is given by

$$z^* = \frac{r + \delta}{Q}. \tag{9}$$

In sum, the entrepreneurs who draw $z_i \geq z^*$ produce. Since the debt constraints bind the active entrepreneurs, they become borrowers.

On the other hand, the financial constraints are ineffective for the entrepreneurs who draw $z_i < z^*$. However, in the competitive final good market, the firms with $z_i < z^*$ cannot compete with the firms whose efficiency is z^*. Thus the entrepreneurs who own the firms with $z_i < z^*$ give up production and become lenders.

Notice that each entrepreneur's budget constraint is rewritten as

$$\dot{a}_i = ra_i + \hat{\pi}_i\left(z_i, r\right)k_i - c_i,$$

where $\hat{\pi}_i = Qz_i - r - \delta$. As to the inactive entrepreneurs, $\hat{\pi}\left(z_i, r\right) = 0$ so that their optimal consumption is $c_i^e = \rho_e a_i$. For the active entrepreneurs, $\pi_i\left(z_i, r\right)$ is stochastic. Itokhoki and Moll (2014) confirm that the active entrepreneurs' optimal consumption decisions are also given by $c_i = \rho_e a_i$.[2] Hence,it holds that

$$c_i^e = \rho_e a_i \quad \text{for all } i \in [0, 1]\,. \tag{10}$$

3 Aggregation

Notice that the debt constraint, $k_i = \theta a_i$, is effective only for the entrepreneurs who draw $z_i \geq z^*$. Therefore, using (4), we aggregate the effective debt constraints to obtain

$$K = \theta\left(z^*\right)^{-\psi}A,$$

where $K = \int_0^1 k_i di$ and $A = \int_0^1 a_i di$. This gives an alternative expression of z^*:

$$z^* = \left(\frac{\theta A}{K}\right)^{\frac{1}{\psi}}. \tag{11}$$

Keeping in mind that z_i is assumed to be iid, we may aggregate the production functions in such a way that

$$Y = \int_0^1 \int_{z \leq z^*} y_i dF\left(z\right)di = Q\int_0^1 \int_{z \geq z^*} zk_i dF\left(z\right)k_i di.$$

From (4) the above is expressed as

$$Y = \frac{\psi}{\psi - 1}Qz^*K.$$

[2] They set up the Hamilton-Jacobi-Bellman equation for the entrepreneur's' optimization problem and specify the value and policy functions by guess and verify method.

Thus from (11) the reduced form of the aggregate production function can be written as

$$Y = \frac{\psi}{\psi - 1} Q \left(\frac{\theta A}{K} \right)^{\frac{1}{\psi}} K, \tag{12}$$

implying that it is homogeneous of degree one with respect to K and A.

As to the behavior of the aggregate net worth, we see that A changes according to

$$\dot{A} = rA + \Pi - C_e, \tag{13}$$

where $\Pi = \int_0^1 \pi_i di$ and $C_e = \int_0^1 c_i^e di$. Here, in view of (9) , it holds that

$$\Pi = Y - (r + \delta) K = Y - Qz^* K.$$

As a result, combining (1) and (13) , we obtain the market equilibrium condition for the final goods:

$$Y = \dot{K} + \delta K + C_e + C_r, . \tag{14}$$

Finally, the equilibrium condition for the financial market is given by

$$K = A + B. \tag{15}$$

4 Balanced-Growth Equilibrium

Aggregating (10) yields $C_e = \rho_e A$. Therefore, using (1) , (14) and (15) , we derive the aggregate dynamic equations of K as follows:

$$\frac{\dot{K}}{K} = \frac{\psi}{\psi - 1} Q \left(\frac{\theta A}{K} \right)^{\frac{1}{\psi}} - \rho_e \frac{A}{K} - \rho_r \frac{K - A}{K} - \delta. \tag{16}$$

Noting that $r = Qz^* - \delta$ and $\Pi = Y - Qz^* K$, we find that the total net worth of the entrepreneurs follows

$$\frac{\dot{A}}{A} = Q \left(\frac{\theta A}{K} \right)^{\frac{1}{\psi}} + \frac{1}{\psi - 1} Q \left(\frac{\theta A}{K} \right)^{\frac{1}{\psi} - 1} - \rho_e - \delta. \tag{17}$$

Now define $x = A/K$. Then (16), (17) and $\dot{x}/x = \dot{A}/A - \dot{K}/K$ present:

$$\frac{\dot{x}}{x} = \frac{1}{\psi - 1} Q \left(\theta x\right)^{\frac{1}{\psi} - 1} - \frac{1}{\psi - 1} Q \left(\theta x\right)^{\frac{1}{\psi}} + (\rho_r - \rho_e)(1 - x), \tag{18}$$

which summarizes the dynamic behavior the aggregate economy. Since the initial level of $x\,(= A/K)$ is historically given, the economy has transition dynamics even if the production technology exhibits an Ak property.

The balanced-growth equilibrium is established when $\dot{K}/K = \dot{A}/A = \dot{Y}/Y$ so that r and x stay constant over time. From (18) the steady-state condition is given by

$$\frac{1}{\psi - 1} Q \left(\theta x\right)^{\frac{1}{\psi} - 1} + \left(\rho_r - \rho_e\right)\left(1 - x\right) = \frac{1}{\psi - 1} Q \left(\theta x\right)^{\frac{1}{\psi}}. \tag{19}$$

It is easy to see that if $\rho_r \geq \rho_e$,then equation (19) has a unique positive solution. Hence, if the steady-state level of x satisfies $x \leq 1$, we obtain a unique, feasible balanced-growth equilibrium. It is also seen that in this case the steady-state solution of (18) is globally stable. However, if $\rho_r < \rho_e$, the graph of the left-hand-side in (19) is U-shaped, which means that (19) would have dual solutions. In the following, we focus on the case where rentiers are less patient than entrepreneurs: $\rho_r \geq \rho_e$.

Note that if there is no rentier, then borrowing and lending are conducted among the entrepreneurs alone, so that $A = K$. In this case, (11) shows that the cutoff level is fixed at $z^* = \theta^{1/\psi}$. This means that from (17) the economy always stays on the balanced-growth path and the long-term growth rate of aggregate income is given by

$$g = \frac{\psi}{\psi - 1} \theta^{\frac{1}{\psi}} Q - \rho_e - \delta.$$

Hence, a weaker financial constraint (a higher value of θ) yields a higher rate of long-term growth. This result is intuitively plausible, because a rise in θ increases the cutoff level of efficiency, $z^* = \theta^{1'\psi}$, implying that production is taken place by the firms with higher levels efficiency. This positive impact enhances income expansion.

In the general setting where rentiers also save, the balanced-growth rate is given by

$$g = \frac{\dot{K}}{K} = \frac{\psi}{\psi - 1} Q \left(\theta x^*\right)^{\frac{1}{\psi}} + \left(\rho_r - \rho_e\right) x^* - \rho_r - \delta,$$

where x^* denotes the steady-state value of x that satisfies (19). First, assume that both rentiers and entrepreneurs have the same discount rate: $\rho_r = \rho_e$.Then the balanced-growth rate is

$$g = \frac{\psi}{\psi - 1} Q \left(\theta x^*\right)^{\frac{1}{\psi}} - \rho_r - \delta.$$

and (19) becomes

$$\frac{1}{\psi - 1} Q \left(\theta x^*\right)^{\frac{1}{\psi} - 1} = \frac{1}{\psi - 1} Q \left(\theta x^*\right)^{\frac{1}{\psi}},$$

which leads to $\theta x^* = 1$. As a result, a change in θ will not affect the long-term growth rate of income.

When $\rho_r > \rho_e$, in view of (19), we find that an increase in θ depresses the steady state value of x^*. Since the effect of a rise in θ on the balanced-growth rate is analytically ambiguous, we examine numerical examples. Our benchmark parameter specification is as follows:

$$Q = 0.08, \quad \rho_r = 0.05, \quad \rho_e = 0.02, \quad \delta = 0.1.$$

Given these parameter values, when $\psi = 1.5$ and $\theta = 1.5$, the steady-state level of the relative wealth share is $x^* = 0.704$ and the balanced growth rate is $g = 0.1208$. If θ increases up to 6.0, then $x^* = 0.276$ and $g = 0.127$. Therefore, when the shape parameter, ψ, is relatively small (so the degree of heterogeneity in production efficiency is relatively high), a rise in θ significantly reduces x but its impact on the balanced-growth rate is small. On the other hand, if $\psi = 6.0$ so that firm heterogeneity is relatively low, then $x^* = 0.869$ and $g = -0.032$ if $\theta = 1.5$, while $x^* = 0.547$ and $g = -0.016$ if $\theta = 6.0$. Those examples indicate that under our specification of parameter values, an increase in the efficiency of financial market would have a small impact on long run-growth if the firm heterogeneity is high, whereas it has a relatively large effect on growth in the presence of a low level of firm heterogeneity.

5 Remarks

This paper constructs a simple model of endogenous growth with financial frictions and firm heterogeneity. We have shown that even if production technology of an individual firm is a simple Ak type, the aggregate economy exhibits transition dynamics and the balanced-growth rate of the economy depends on the macroeconomic distribution of wealth between rentiers and entrepreneurs. Our model is simple enough to discuss various issues in growth economics such as long-run impacts of fiscal policy and the relation between bubbles and TFP in a tractable manner.[3] It is also to be pointed out that our discussion can be extended to a more general setting where there are workers and entrepreneurs. In this case, production activities use labor as well as capital and labor supply is endogenously determined. Chen and Mino (2014) investigate such a generalized model.

[3] For example, our model would present an alternative approach to Jaimobich and Rebelo (2012) on taxation and growth and to Miao and Wang (2012) on the relation between bubbles and TFP. Those authors employ more complex models of firm heterogeneity than ours.

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