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Conformism and Wealth Distribution*

Kazuo Mino[†] and Yasuhiro Nakamoto[‡]

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Abstract

This paper explores the role of consumption externalities in a neoclassical growth model in which households have heterogeneous preferences. We find that the degree of conformism in consumption held by each household significantly affects the speed of convergence of the aggregate economy as well as the patterns of wealth distribution in the steady state equilibrium. In particular, a higher degree of consumption conformism accelerates the convergence speed of the economy towards the steady state. We also reveal that in an economy with a high degree of conformism, the pattern of initial distribution of wealth tends not to be sustained in the long run.

Keywords: consumption externalities, heterogeneous agents, wealth distribution

JEL Classification Code: D31, E13, E21, O40

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1 Introduction

It has long been recognized that social comparison is one of the central features of human behavior. In recent years, a number of experimental studies challenge to investigate whether social comparison affects individual well-being. For example, Fliessbach et al. (2007) examine the impact of social comparison on brain activity using functional magnetic resonance imaging (fMRI), showing that not only the absolute level of payment but also relative level of payment similarly affect brain activity. In their paper, it is considered that neurophysiological evidence supports the importance of social comparison in the human brain. Using survey-experimental methods, Alpinar et al (2005) show that, on average, both absolute and relative consumption matter for individual well-being, and conclude that most individuals are interested in others' consumption of particular goods such as car and housing.

Along with the development in the neurosciences and behavioral economics, there has been a renewed interest in the role of consumption externalities in macroeconomic dynamics. The basic assumption of this literature is that consumers' felicity depends not only on their private consumption but also on the average consumption in the economy at large. The presence of such a psychological external effect may alter saving behaviors of consumers and thus dynamic property of the model economy.¹ The existing studies have inspected the effects of consumption externalities in a variety of topics. A sample includes asset pricing (Abel 1990 and Galí 1994), income taxation (Ljungqvist and Uhlig 2000 and Fisher and Hof 2000), equilibrium efficiency (Liu and Turnovsky 2005, Nakamoto 2009 and Arrow and Dasgupta 2009), belief-driven business cycles (Alonso-Carrera, et al. 2008, Chen and Hsu 2007, Chen et al. 2013 and 2014, and Weder 2000) and long-term economic growth (Carroll et al. 1997 and 2000, and Harbaugh 1996).²

All of the existing macroeconomic studies mentioned above use representative-agent mod-

¹Other influential studies on economic analyses of consumption externalities include Carlsson et al. (2007), Clark et al. (2008), Dupor and Liu (2003), Easterlin (2001), Fank (2005) and Luttmer (2005).

²Some of the existing studies such as Ljunavust and Uhlig (2000) and Carroll et al. (1997 and 2000) assume the external habit formation in which the benchmark consumption is given by a weighted average of past levels of the average consumption in the economy. Unlike the internal habit formation, consumers consider that the benchmark consumption is not affected by their own consumption behavior under the external habit formation hypothesis. Thus this assumption represents consumption externalities with time delay rather than (internal) habit formation under which each agent takes its past consumption into account when deciding its optimal saving plans.

els. In the representative-agent economy, the social average consumption coincides with the level of private consumption. Therefore, despite the importance of distinguishing divergent degree of conformism among the agents, the existing studies employing the representative-agent models with consumption externalities fail to capture the underlying economical basis of social comparison in a satisfactory manner. In contrast to the mainstream literature, this paper explores the role of social comparison in a neoclassical growth model with heterogeneous agents.³ We assume that each household may have a different preference structure as well as a different level of wealth. In particular, we focus on how the heterogeneity in the degree of external effect in consumption (the degree of consumers' conformism) affects wealth distribution and the convergence speed of the economy. In our setting, externalities of consumption activities yield more complex outcomes than those in the representative-agent economy. This is mainly because in our model the presence of consumption externalities may have distributional effects that are inevitably absent in the representative agent modelling⁴.

In this paper we use a standard Ramsey model with fixed labor supply, so that the steady state of the aggregate economy is the same as that of the model without consumption externalities. Hence, we are mainly concerned with the transitional dynamics of the aggregate economy as well as with the steady-state distribution of income and wealth. First, we investigate how the consumption externalities with heterogeneous preferences affect the speed of convergence of the macroeconomy. It has been well-known that a higher degree of consumption externalities generally accelerates speed of convergence of the representative agent economy. We find the speed of converges increases with the aggregate level of consumer conformism in our heterogeneous-agent model as well. Furthermore, we reveal that the speed of convergence also depends on the pattern of wealth distribution.

The second issue we address is to consider how the presence of consumption externalities affect long-run distribution of wealth among the households. In the representative-agent economy, the initial pattern of distribution tend to be preserved in the long-run equilibrium

³Many behavioral economics studies have emphasized that behavior of social comparison is heterogeneous among consumers depending on the agents' characteristics such as income, age, race, gender, family status, education, occupation and urbanity: see, for example, Burns (2006), Hewstone et al. (2002), Maurer and Meier (2008), Mullen et al. (1992), and Rubin and Willis (2002).

⁴Some authors such as Abel (2005), Mino (2008) and Alvarez-Cuadrado and Long (2011) introduce consumption external effects into the overlapping generations models where heterogeneity of agents inevitably exists.

even in the presence of social comparison behavior of households. In our setting, however, the long-run distribution of wealth is highly sensitive to the degree of external effect perceived by each household: depending on the degree of conformism held by each agent, the long-run wealth distribution may be substantially different from the initial distribution.

When dealing with the two issues mentioned above, we particularly pay attention to the two alternative specifications of utility function that have been often employed in the literature: the utility functions with subtractive external effects and with multiplicative external effects. We find that these alternative specifications may yield different outcomes and implications both for the aggregate dynamics of the economy and for long-run distribution of income and wealth.

It is to be noted that our study is closely related to García-Peñalosa and Turnovsky (2008). These authors also examine a heterogeneous-agent model of neoclassical growth with consumption externalities. García-Peñalosa and Turnovsky (2008) consider a model with variable labor supply. In this respect, their model is more general than our setting with fixed labor supply. On the other hand, they assume that each type of agent has the same form of utility function that satisfies quasi-homotheticity, so that the equilibrium dynamics of the aggregate economy is completely independent of wealth distribution. As a result, García-Peñalosa and Turnovsky (2008) can focus on the distribution dynamics under a given pattern of macro economic dynamics.⁵ As emphasized above, since our model assumes that the inclusion of the external effects is more general in the sense that the utility function is not necessarily quasi-homothetic, the behavior of the aggregate economy cannot be separated from personal wealth distribution.⁶

The remainder of this paper is organized as follows. Section 2 sets up the baseline framework. Section 3 characterizes the steady-state equilibrium and explores the relation between consumption externalities and the stationary distribution of wealth. Section 4 discusses transitional dynamics and considers the effects of consumption externalities on the behavior relative wealth. Section 5 concludes.

⁵Their contribution is an extension of García-Peñalosa and Turnovsky (2008) who examine the corresponding model without consumption externalities. See also Caselli and Ventura (2000) for a typical example where behavior of the aggregate economy is independent of income distribution.

⁶Mino and Nakamoto (2009 and 2013) also explore the models with heterogeneous agents and consumption externalities. Since these papers assume that there are only two types of households, the discussion of this paper is far more general than Mino and Nakamoto (2009 and 2013).

2 Baseline Setting

2.1 Production and Consumption

We consider a simple neoclassical growth model with identical firms.⁷ The aggregate production function is assumed to satisfy constant returns to scale with respect to capital and labor, and it is expressed as

$$Y = F(\hat{K}, L) = Lf(K)$$

where Y is output, \hat{K} is capital, L is labor and $K \equiv \hat{K}/L$ denotes capital intensity. The productivity function, $f(K)$, is monotonically increasing, strictly concave in K and satisfies the Inada conditions. In competitive factor and final good markets, the real rent and real wage rate are respectively determined by

$$r = f'(K) = r(K), \quad w = f(K) - Kf'(K) = w(K). \quad (1)$$

Households are assumed to be heterogeneous in the sense that each household has agent-specific preferences and different stock of wealth. The instantaneous utility function of type i household is

$$u^i = u^i(c_i, C_i).$$

Here, c_i denotes private consumption of type i household and C is the average consumption of a set of agents. Namely, C_i is the reference consumption of household i ; the felicity of each household is affected by the average consumption of some specific group of agents. Thus C_i is defined as

$$C_i = \int_{i \in N_i} c_i di,$$

where N_i is a set of agents whose average consumption affects the felicity of type i agent.

In this paper we focus on the simplified case where external effects prevail the entire economy, so that the reference consumption for each group is the average consumption in the economy at large. We also assume that each type of agent is uniformly distributed over the

⁷If there is no external effect in consumption, our analytical framework is similar to the neoclassical growth model with heterogeneous agents examined by Becker (1980), Chatterjee (1994), Foellmi (2008) and Sorger (2002).

region $[0, 1]$, implying that $C_i = C$ for all $i \in [0, 1]$, where the average consumption is defined as

$$C = \int_0^1 c_i di. \quad (2)$$

Note that the mass of households is normalized to unity, so that C denotes the aggregate consumption as well.

The i -th agent maximizes a discounted sum of utilities

$$U^i = \int_0^\infty e^{-\rho t} u^i(c_i, C) dt$$

subject to flow budget constraint

$$\dot{a}_i = r a_i + w l_i - c_i, \quad (3)$$

together with a given initial holding of wealth $a_i(0)$ and the non-Ponzi game condition such that

$$\lim_{t \rightarrow \infty} \exp\left(-\int_0^t r_s ds\right) a_i \geq 0. \quad (4)$$

When solving this problem, the household takes the entire sequence of the reference consumption, $\{C(t)\}_{t=0}^\infty$, as given.

Denoting the (private) utility price of capital by q_i , the optimization conditions give the following:

$$u_1^i(c_i, C) = q_i, \quad (5)$$

$$\dot{q}_i = q_i(\rho - r), \quad (6)$$

as well as the transversality condition:

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_i a_i = 0. \quad (7)$$

2.2 Market Equilibrium Conditions

We assume that each household supplies one unit of labor in each moment of time so that $l_i = 1$. Since the mass of households is unity, the aggregate labor is also $L = 1$. The net wealth of this economy is the aggregate capital stock, and thus the equilibrium condition of

the asset market is given by

$$K = \int_0^1 a_i di.$$

Since K is only real asset, we may assume that households directly own real capital, so that we set $a_i = k_i$ in the subsequent discussion.

Finally, the equilibrium condition of the final good market is

$$Y = \dot{K} + C. \quad (8)$$

For notational simplicity, we ignore capital depreciation.

2.3 Conformism and Consumption Behavior

Since we restrict our attention to the standard case where $N_i = [0, 1]$ for all i , it holds that $C_i = C$ for all i , so that (5) and (6) yield

$$\dot{c}_i = -\frac{u_1^i(c_i, C)}{u_{11}^i(c_i, C)}(r - \rho) - \frac{u_{12}^i(c_i, C)}{u_{11}^i(c_i, C)}\dot{C}.$$

We express this equation as

$$\dot{c}_i = \phi_i(c_i, C)(r - \rho) + \Lambda_i(c_i, C)\dot{C}, \quad (9)$$

where

$$\phi_i(c_i, C) = -\frac{u_1^i(c_i, C)}{u_{11}^i(c_i, C)} > 0, \quad (10a)$$

$$\Lambda_i(c_i, C) = -\frac{u_{12}^i(c_i, C)}{u_{11}^i(c_i, C)}. \quad (10b)$$

In the above, $1/\phi_i(c_i, C)$ represents the degree of absolute risk aversion of type i household. Following Gollier (2004), we call $\Lambda_i(c_i, C)$ the *degree of conformism* of type i household. This function shows how the private consumption responds to a change in the average consumption to keep the marginal utility of private consumption constant. If $\Lambda_i > 0$, the household i is a conformist in the sense that she changes her own consumption in the same direction of the change in the average consumption. In contrast, if $\Lambda_i(c_i, C) < 0$, the household changes her consumption in the opposite direction. Moreover, when $\Lambda_i(c_i, C) > 1$, the household is an

over-conformist, because the household changes her consumption more than a change in the average consumption to keep her marginal utility of private consumption unchanged. In this paper we focus on the case each household is a conformist, so that we are interested in the case that $\Lambda_i(c_i, C) \geq 0$ for all $i \in [0, 1]$ and $t \geq 0$.

In the existing macroeconomic studies on consumption externalities, the following two types of specification of the utility function have been frequently employed.

(i) *Subtractive External Effects*

One of the well-used formulations of external effects is the subtractive form of externalities under which the instantaneous utility function is specified as⁸

$$u^i(c_i, C) = v_i(c_i - \eta_i(C)), \quad i \in [0, 1], \quad (11)$$

where $v^i(\cdot)$ is monotonically increasing and strictly concave in ‘net’ consumption $z_i \equiv c_i - \eta_i(C)$, while $\eta_i(\cdot)$ is monotonically increasing function of C . Namely, the felicity generated by private consumption plus the negative effect of social comparison generate the utility of consumption of each agent. We can see

$$\phi_i(c_i, C) = -\frac{v_i'(z_i)}{v_i''(z_i)}(> 0), \quad \Lambda_i(C) = \eta_i'(C>(> 0).$$

The specification of this type gives two characteristics. First, the inverse of the degree of absolute risk aversion, $\phi_i(c_i, C)$ depends on not only the private consumption but also the social consumption. Second, the degree of conformism depends on the average consumption alone. We may call $\Lambda_i(C)$ the *separable conformism*. For instance, we can give a simple specification of this type of utility function:

$$u^i(c_i, C) = \frac{(c_i - \eta_i(C))^{1-\beta_i}}{1-\beta_i}, \quad \beta_i \neq 1, \quad \beta_i > 0, \quad (12)$$

where

$$\phi_i(c_i, C) = \frac{c_i - \eta_i(C)}{\beta_i}(> 0), \quad \Lambda_i = \eta_i'(C).$$

⁸See Campbell and Cochrane (1999).

Notice that the function ϕ_i is linear in the net consumption $c_i - \eta_i(C)$. If the external effect is also linear in C such that $\eta_i(C) = \epsilon_i C$, then the degree of conformism is given by a parameter $\epsilon_i (> 0)$.

Under the separable conformism, the agent i is the over-conformist if $\eta'_i(C) > 1$, which means that the agent i becomes the over-conformist if the negative external effect is large enough. In particular, taking account of a linear function $\eta_i(C) = \epsilon_i C$, the agents who have $\epsilon_i > 1$ are categorized as the over-conformists at any time. In contrast, suppose that $\eta_i(C)$ is not linear function. Then, since the steady-state level of aggregate consumption is determined by the fixed rate of time preference as confirmed later, the condition of over-conformist can be rewritten as $\eta'_i(C^*(\rho)) > 1$ at the steady state.

(ii) *Multiplicative External Effects*

The other formulation of utility function often used in the literature is the multiplicative form of externalities.⁹ In this formulation the utility function is given by

$$u^i(c_i, C) = v_i(c_i) \eta_i(C), \quad i \in [0, 1], \quad (13)$$

where $v'_i(c_i) > 0$, $v''_i(c_i) < 0$ and $\eta'_i(C) > 0$. Given this functional form, we obtain

$$\phi_i(c_i) = -\frac{v'_i(c_i)}{v''_i(c_i)} (> 0), \quad \Lambda_i(c_i, C) = \phi_i(c_i) \frac{\eta'_i(C)}{\eta_i(C)} (> 0).$$

Note that in contrast to the case of subtractive external effects, if the external effects are introduced in the multiplicative form, the absolute risk aversion depends only on the private consumption, while the degree of conformism is affected by private as well as social levels of consumption (the case of *nonseparable conformism*). A typical example of the utility function with the multiplicative external effects is:

$$u^i(c_i, C) = \frac{(c_i C^{-\theta_i})^{1-\gamma_i}}{1-\gamma_i}, \quad \gamma_i > 0, \quad \gamma_i \neq 1, \quad 0 < \theta_i < 1. \quad (14)$$

⁹For instance, this type of utility function is given in Gali (1994) and Carroll et al (1997).

In this specification we obtain

$$\phi_i(c_i, C) = \frac{c_i}{\gamma_i}, \quad \Lambda_i(c_i, C) = \left(1 - \frac{1}{\gamma_i}\right) \theta_i \frac{c_i}{C},$$

where $\Lambda_i > 0$ leads to the assumption that $\gamma_i > 1$. Thus the degree of individual conformism depends on the the intertemporal elasticity of private consumption, $1/\gamma_i$, the individual degree of external effect, θ_i , as well as on the private consumption relative to the social average, c_i/C . Unlike the specified utility function (12), notice that even if the preference parameters are identical ($\gamma_i = \gamma$ and $\theta_i = \theta$ for all i), the degree of individual conformism may differ each other unless $c_i = C$ for all i .

Taking account of the non-separable conformism, the agent i is the over-conformist if $\eta'_i(C)/\eta_i(C) > 1/\phi_i(c_i)$. Therefore, in the case of multiplicative utility function, the degree of absolute risk aversion $1/\phi_i(c_i)$ affects the appearance of over-conformist; that is, even if the external effect $\eta'_i(C)/\eta_i(C)$ is not large enough, the agent may be the over-conformist if the degree of absolute risk aversion is small. In this paper we will restrict our attention to the case where households are not over-conformists on average, so that the aggregate (average) level of the degree of conformism does not exceed to the unity as in the below.

2.4 Equilibrium Dynamics

Using (9), we see that the average (aggregate) consumption follows:

$$\dot{C} = (r - \rho) \int_0^1 \phi_i(c_i, C) di + \left(\int_0^1 \Lambda_i(c_i, C) di \right) \dot{C},$$

which leads to

$$\dot{C} = \Delta (r - \rho), \quad \Delta \equiv \frac{\int_0^1 \phi_i(c_i, C) di}{1 - \int_0^1 \Lambda_i(c_i, C) di}, \quad (15)$$

where Δ/C represents the inverse of the elasticity of intertemporal substitution in social consumption. In the following, we assume that the average degree of conformism of the society does not exhibits over-conformism so that

$$\int_0^1 \Lambda_i(c_i, C) di < 1, \quad (16)$$

implying that Δ has a positive value. Under the assumption (16), applying for the separable conformism such as a linear form $\eta_i(C) = \beta_i C$ in (12) yields $\int_0^1 \beta_i di < 1$, which shows that the condition (16) gives a restriction on the average degree of conformism; instead, using (14) with $\gamma_i = \gamma$, we obtain the assumption $\int_0^1 \theta_i c_i di / C < \gamma / (\gamma - 1)$, which consists of the degrees of conformism θ_i , the level of private consumption as well as the aggregate consumption and the degree of absolute risk aversion. Equation (15) shows that, other things being equal, a higher degree of average level of conformism makes the average consumption more sensitive to a change in the real interest rate, r .

Substituting (15) into (9), we find that the consumption of individual household follows

$$\dot{c}_i = [\phi_i(c_i, C) + \Lambda_i(c_i, C) \Delta] (r - \rho). \quad (17)$$

This expression means that when the household of type i has a higher degree of conformism, $\Lambda^i(c_i, C_i)$, her private consumption is more sensitive to a change in the real interest rate.

From (8) the dynamic behavior of the average capital follows

$$\dot{K} = f(K) - C. \quad (18a)$$

As a consequence, a complete dynamic system of the entire economy consists of (17), (18a),

$$\dot{k}_i = rk_i + w - c_i, \quad (18b)$$

together with (1), (2), and a given initial level of capital distribution among the households.

The steady-state levels of average consumption and capital stock, C^* and K^* , are uniquely determined by

$$f(K^*) = C^*, \quad (19a)$$

$$f'(K^*) = \rho. \quad (19b)$$

The above steady-state conditions demonstrate that distribution of wealth and the presence of consumption externalities fail to affect the steady-state levels of average variables as in the representative-agent model with the fixed labor supply. In addition, when all the households are conformists so that $\Lambda_i(c_i, C) > 0$ for all i , and if the the average degree of conformism

satisfies (16), then the average Euler equation exhibits the familiar pattern of dynamics: the average consumption increases (decreases) when the average capital is lower (higher) than its steady state level, K^* . Therefore, K and C follow a stable saddle path converging to the steady state given above.

3 Speed of Convergence

3.1 Convergence Speed of the Aggregate Economy

We first examine local dynamics of the aggregate system around the steady state equilibrium. The linear approximated system of (15) and (18a) at the steady state consists of the following dynamic equations

$$\begin{aligned}\dot{K} &= \rho(K - K^*) - (C - C^*), \\ \dot{C} &= \Delta^* f''(K^*)(K - K^*),\end{aligned}$$

where Δ^* denotes the steady state level of Δ given by the following:

$$\Delta^* = \frac{\int_0^1 \phi_i(c_i^*, C^*) di}{1 - \int_0^1 \Lambda_i(c_i^*, C^*) di}$$

and c_i^* denotes the steady-state level of the individual consumption.

Noting that $K^* = f'^{-1}(\rho)$ and $C^* = f(f'^{-1}(\rho))$, we can rewrite $K^* = K^*(\rho)$, $C^* = C^*(\rho)$ and $w^* = w^*(\rho)$. Therefore, the steady-state value of c_i satisfies

$$c_i^* = \rho k_i^* + w^*(\rho). \quad (20)$$

Furthermore, we find that the stable root of the above system is

$$\mu_s = \frac{1}{2} \left[\rho - (\rho^2 - 4\Delta^* f''(K^*(\rho)))^{1/2} \right] < 0. \quad (21)$$

The variable Δ^* is given by:

$$\Delta^* = \frac{\int_0^1 \phi_i^* di}{1 - \int_0^1 \Lambda_i^* di}, \quad (22a)$$

where

$$\phi_i^* = \phi_i(\rho k_i^* + w^*(\rho), C^*(\rho)), \quad \Lambda_i^* = \Lambda_i(\rho k_i^* + w^*(\rho), C^*(\rho)). \quad (22b)$$

Since the absolute value of the stable root represents the speed of convergence on the aggregate economy on the stable saddle path, we immediately see that the economy with a higher degree of average conformism, and hence a higher value of Δ^* exhibits a higher speed of convergence towards the steady state. Concretely, the speed of convergence is faster as the positive value of $\int_0^1 \Lambda_i^* di$ approaches the unity and the positive value of $\int_0^1 \phi_i^* di$ is larger. Furthermore, and more importantly, because of the heterogeneity of preferences, the steady-state distribution of capital itself affects the value of Δ^* ; hence, in next subsection, we mention the relationship between the speed of convergence and the wealth distribution.

Dynamic behaviors of individual consumption and wealth are described by

$$\begin{aligned} \dot{k}_i &= rk_i + w - c_i = f'(K)(k_i - K) + f(K) - c_i, \\ \dot{c}_i &= [\phi_i(c_i, C) + \Lambda_i(c_i, C)\Delta](f'(K) - \rho). \end{aligned}$$

On the stable saddle path of the aggregate system, it holds that $C - C^* = (\rho - \mu_s)(K - K^*)$. Hence, the approximated behavior of individual capital, individual consumption and the aggregate capital respectively follow

$$\begin{aligned} \dot{k}_i &= \rho(k_i - k_i^*) - (c_i - c_i^*) + f''(K^*)(k_i^* - K^*)(K - K^*), \\ \dot{c}_i &= \Phi_i^* f''(K^*)(K - K^*), \\ \dot{K} &= \mu_s(K - K^*). \end{aligned}$$

In the above,

$$\Phi_i^* = \phi_i^* + \Lambda_i^* \Delta^* > 0. \quad (23)$$

Note that the stable root of this system is still μ_s , which means that on the stable saddle path of the entire economy each relation between individual capital (or individual consumption)

and the aggregate capital satisfies

$$k_i - k_i^* = \frac{\frac{\Phi_i^* f''(K^*)}{\mu_s} - f''(K^*)(k_i^* - K^*)}{\rho - \mu_s} (K - K^*), \quad (24)$$

$$c_i - c_i^* = \frac{\Phi_i^* f''(K^*)}{\mu_s} (K - K^*). \quad (25)$$

Therefore, on the approximated saddle path both individual consumption and capital move into the same direction as the aggregate capital changes. In addition, it is seen that, other things being equal, a higher level of individual conformism (a higher value of $\Lambda_i(c_i, C)$) raises the responses of c_i and k_i to a change in the aggregate capital.

To sum up, we have seen the following result as to the local dynamics of the economy:

Proposition 1 (i) *The speed of convergence of the aggregate economy increases with the degree of average conformism in the economy at large; and (ii) the speed of convergence of capital and consumption of each consumer increases with her own degree of conformism.*

3.2 Wealth Distribution and Convergence Speed: HARA family

As García-Pñalosa and Turnovsky (2008) demonstrate, if each household has the same preference structure that satisfies quasi-homotheticity in private as well as social consumption, then the aggregate dynamics is independent of income distribution among agents even in the presence of consumption externalities. Therefore in their model, the speed of convergence of the aggregate economy is not affected by the pattern of wealth distribution. Since in our setting the instantaneous felicity function is not necessarily quasi-homothetic, patterns of income distribution may affect the aggregate behavior of the economy so that the convergence speed depends on income distribution as well.

It has been well-known that with HARA preferences, the marginal utility of consumption is proportional to a power of a linear function of the consumption level, which gives an important property of utility function that the absolute risk aversion is a linear function in private consumption.¹⁰ As a result, it can be seen that if the degrees of absolute risk aversion among agents are homogeneous, the ϕ_i function which holds for the individual can be reduced to a ϕ function in aggregate consumption.

¹⁰See Bertola et al (2006).

(i) *Subtractive External Effects*

As discussed in Section 2.3, in this case $\phi_i = -v'_i(c_i - \eta_i(C))/v''_i(c_i - \eta_i(C))$ and $\Lambda_i = \eta'_i(C)$. If the households have identical preference $v_i(\cdot) = v(\cdot)$ and $\eta_i(\cdot) = \eta(\cdot)$, Δ^* can be written as

$$\Delta^* = \frac{\int_0^1 \phi(c_i^* - \eta(C^*)) di}{1 - \eta'(C^*)}. \quad (26)$$

In particular, if the utility function takes a general formation of HARA family, we have the additively linear-separable function in $\phi(c_i^* - \eta(C^*)) = \phi(c_i^*) - \phi(\eta(C^*))$ where note that $\phi(\eta(C^*))$ is common for all agents. Since $\phi(c_i^*)$ is linear in the private consumption due to HARA preference, the integral $\int_0^1 \phi(c_i^*) di$ can be reduced to a function in the aggregate consumption. As a result, Δ^* is not affected by the dispersion of individual wealth. In other words, the wealth distribution does not affect the speed of convergence as long as the utility function is HARA family.

Next, we loosen the restriction on the homogeneous conformism. That is, we assume that the function $v_i(\cdot)$ is identical for all i and HARA family, but the external effect $\eta_i(C)$ is different from each other. However, even if this were the case, the additive separability $\phi(c_i^* - \eta(C^*)) = \phi(c_i^*) - \phi(\eta_i(C^*))$ still holds, meaning that $\phi(c_i^*)$ can be reduced to a function in the aggregate consumption. Therefore, even if the heterogeneity of conformism leads to a dispersion of wealth, it is not the key element to have the impacts of wealth distribution on the speeds of convergence. To see it, assume that $\beta_i = \beta$ for all agents i in (12). The Δ^* is:

$$\Delta^* = \frac{C^*(\rho) \left(1 - \int_0^1 \eta_i(C^*(\rho))/C^*(\rho) di\right)}{\beta \left(1 - \int_0^1 \eta'_i(C^*(\rho)) di\right)}.$$

We can easily see that the value of Δ^* is determined irrespective of wealth dispersion, that is, the wealth distribution does not affect the speed of convergence even if the external effect is heterogeneous. In particular, considering that $\eta_i(C) = \epsilon_i C$, it holds that $\Delta^* = C^*/\beta$.

Instead, if not only the external effect but also the utility function $v_i(\cdot)$ are heterogeneous, we can obtain

$$\Delta^* = \frac{\int_0^1 \phi_i(c_i^*) di - \int_0^1 \phi_i(\eta_i(C^*)) di}{1 - \int_0^1 \eta'_i(C^*) di}.$$

Even if $\phi_i(c_i^*)$ is linear in private consumption, the integral $\int_0^1 \phi_i(c_i^*) di$ cannot be rewritten

as an aggregate variable. Therefore, the change in wealth distribution affects the speeds of convergence. More precisely, using the specified utility function (12), we obtain:

$$\Delta^* = \frac{\int_0^1 (c_i^* - C^*(\rho)\epsilon_i)/\beta_i di}{1 - \int_0^1 \epsilon_i di}. \quad (27)$$

Intuitively, if relatively wealthier households have the greater degrees of conformism $\Lambda_i = \epsilon_i$ and the inverse of absolute risk aversion $\phi_i = (c_i^* - C^*(\rho)\epsilon_i)/\beta_i$, the speed of convergence is relatively high because they like to save and dislike to consume.

To sum up, we have shown:

Proposition 2 *Suppose that the utility function has the subtractive form of consumption externalities and that it is in the HARA class. The wealth distribution does not have any impacts on the speeds of convergence if the degrees of conformism are heterogeneous alone.*

Finally, we mention the case that $v_i(\cdot)$ is an additively separable formation, that is, $u^i(c_i, C) = v_i(c_i) - \eta_i(C)$. In this case, $\Delta^* = \int_0^1 \phi_i(c_i^*) di$, so that the external effect in consumption will not affect the behavior of the aggregate variables.

(ii) *Multiplicative External Effects*

Turning our interests into the utility function with multiplicative external effects, we find that ϕ_i depends on c_i alone and Λ_i consists of c_i and C . First, assume that the households have identical preference so that $u(c_i, C) = v(c_i)\eta(C)$ for all $i \in [0, 1]$. Given this functional form, Δ^* is expressed as

$$\Delta^* = \frac{\int_0^1 \phi(c_i^*) di}{1 - [\eta'(C^*)/\eta(C^*)] \int_0^1 \phi(c_i^*) di}. \quad (28)$$

If the utility function belongs to the HARA family such as (14), then $\phi(c_i)$ is linear in the private consumption c_i , which implies that the private consumption can be reduced to the aggregate consumption. That is, since Δ^* depends only on C^* , the aggregate dynamics is independent of wealth distribution.

Next, consider the case that only external effect is heterogeneous for each household, i.e.

$u^i(c_i, C) = v(c_i) \eta_i(C)$. In that case, Δ^* is

$$\Delta^* = \frac{\int_0^1 \phi(c_i^*) di}{1 - \int_0^1 \phi(c_i^*) [\eta_i'(C^*) / \eta_i(C^*)] di}. \quad (29)$$

In this case, even if the absolute risk aversion is identical among the agents (i.e., $\phi_i = \phi(c_i^*)$) and furthermore the ϕ function is linear in private consumption due to HARA family, the heterogeneity of conformism allows us to see the impact of wealth distribution on the speeds of convergence. This is because the degree of conformism by each agent consists of not only the aggregate consumption but also the individual consumption unlike the subtract external effects. For instance, making use of (14) with $\gamma_i = \gamma$, we obtain

$$\Delta^* = \frac{C^*(\rho)/\gamma}{1 - \frac{(1-1/\gamma) \int_0^1 \theta_i c_i^* di}{C^*(\rho)}}, \quad (30)$$

which shows that when relatively wealthier households have the greater degrees of θ_i , which corresponds to the stronger degrees of conformism, the speed of convergence is relatively fast.

Proposition 3 *Suppose that the utility function has the multiplicative form of consumption externalities and that it is in the HARA class. Because of the heterogeneity of conformism, the wealth distribution affects the speeds of convergence. In particular, a stronger degrees of conformism held by the relatively wealthier households leads to a faster speed of convergence.*

3.3 Non-HARA family

In the last subsection we have assumed that the utility function is in HARA class, so that the $\phi_i(c_i)$ becomes a linear function in private consumption. Furthermore, assuming that $v_i(c_i)$ is homogeneous for all agents, the wealth distribution does not affect the speeds of convergence under subtract form of heterogeneous-external effects; instead, the wealth distribution influences the speeds of convergence under multiplicative form of heterogeneous-external effects. HARA preferences are used in a lot of manuscript due to the convenience of manipulation; however, the relationship between the wealth distribution and the convergence speeds is simplified to some extent. Therefore, in this subsection we assume that the utility function is in non-HARA class. Naturally, the heterogeneity of conformism may have a more complicated

impact.

Suppose that the utility function has the subtractive external effects as in (12) where $v_i(z_i)$ does not belong to HARA class. Furthermore, for simplicity, we assume that the utility function $v_i(z_i) = v(c_i)$ for all i and $\eta_i(C) = \epsilon_i C$, implying that $\phi_i(z_i) = \phi(z_i)$ where $z_i = c_i - \epsilon_i C$. Then Δ^* becomes

$$\Delta^* = \frac{\int_0^1 \phi(z_i) di}{1 - \int_0^1 \epsilon_i di}. \quad (31)$$

If the agents have HARA preferences, $\phi(z_i)$ is a linear function in net consumption z_i , so that the dispersion of wealth does not influence the speeds of convergence as in Proposition 2. Instead, when the utility function is in non-HARA class, $\phi(z_i)$ may be concave or convex, so that $\phi(\cdot)$ cannot be reduced to a function in the aggregate consumption. Therefore, the wealth distribution impacts the speeds of convergence.

Concretely, in view of the second-order stochastic dominance, we see that if $\phi(z_i)$ is monotonically increasing and strictly convex (concave) in z_i , then a more spread distribution of z_i yields a larger (smaller) value of $\int_0^1 \phi(z_i) di$, and hence the value of Δ^* becomes larger (smaller) given $\int_0^1 \epsilon_i di$ in (31); therefore, the convexity (concavity) of $\phi(z_i)$ leads to a faster (slower) speed of convergence.¹¹ In words, given the degrees of conformism, the economy

¹¹Note that in the case that $v^i(z_i) = v(z_i)$ and $\eta_i(C) = \epsilon_i C$, regardless of distribution of net consumption among the households, the social average of net consumption is $Z = C(1 - \int_0^1 \epsilon_i di)$ where $\int_0^1 \epsilon_i di (< 1)$ is an integral of constant parameters. Thus if the consumption distribution is more spread under a given cumulative distribution functions $F(z)$ than that under $G(z)$, it holds that

$$\int_0^z F(x) dx \geq \int_0^z G(x) dx.$$

If $\phi(z)$ is monotonically increasing and convex in z , then the theorem of the second-order stochastic dominance means that the above condition is equivalent to

$$\int_0^z \phi(x) dF(x) dx \geq \int_0^z \phi(x) dG(x)$$

Here we define

$$F(z) = \text{number of agents whose net consumption is higher than } z$$

Since the number of agents is normalized to one, $F(z)$ is considered the cumulative probability distribution of z . We denote consumption of agent i according to the distribution F and G as c_i^F and c_i^G , respectively. Then it holds that

$$\begin{aligned} B_F &= \int_0^1 \phi(c_i^F) di = \int_0^{\max c^F} \phi(z) dF(z), \\ B_G &= \int_0^1 \phi(c_i^G) di = \int_0^{\max c^G} \phi(z) dG(z), \end{aligned}$$

where $\max c^F$ and $\max c^G$ stand for the maximum levels of individual consumption. As a result, given the

with a higher (lower) degree of inequality converges faster, if $\phi(z_i)$ function satisfies convexity (concavity).

Furthermore, turning our interests into the specification $z_i = c_i - \epsilon_i C$, we can give an additional intuition. The relative wealthier (poorer) households have the smaller (greater) degrees of conformism ϵ_i , which leads to a greater (lower) level of net consumption z_i . In that case, the economy becomes more unequal in net consumption. Then, if the preference specified in $\phi(z_i)$ shows convex function in z_i , the economy converges faster. Instead, if the preference is concave function, the opposite relationship holds.

Next, consider that the utility function has the multiplicative external effects of consumption externalities. In particular, we make use of (13) where we assume that $v_i(c_i)$ is homogeneous (i.e., $v_i(c_i) = v(c_i)$). The assumption that $v(c_i)$ is in non-HARA class allows to see that the $\phi(c_i)$ function may be convex or concave. Moreover, supposing that $\eta_i(C)$ is monotonically increasing and twice differentiable, $\eta'_i(C)/\eta_i(C)$ is defined as a linear function in the aggregate consumption, for instance, $\eta'_i(C)/\eta_i(C) = \alpha_i C$ where α_i is a positive parameter. Then, Δ^* is shown by

$$\Delta^* = \frac{\int_0^1 \phi(c_i) di}{1 - C^*(\rho) \int_0^1 \alpha_i \phi(c_i) di}. \quad (32)$$

Intuitively, we can give the explanation as follows. Suppose that $\phi(c_i)$ is convex function. In that case, a more spread distribution of c_i yields a larger value of $\int_0^1 \phi(c_i) di$ and $\int_0^1 \alpha_i \phi(c_i) di$ given the parameter α_i . In particular, when the relatively wealthier households who have a greater value of $\phi(c_i)$ has a greater value of parameter α_i (i.e., a greater degree of conformism), the value of Δ^* tends to be larger and hence the speed of convergence becomes faster. Alternatively, supposing that $\phi(c_i)$ is concave function, the opposite results are applicable.

assumptions we have made, if the distribution of individual consumption following $F(z)$ is more diverse than that following $G(z)$, and if $\phi(z)$ is monotonically increasing and strictly convex in z , then $B_F \geq B_G$. If $\phi_i(z_i)$ is monotonically increasing and strictly concave function for all agents, then the opposite result holds.

4 Transition Dynamics

4.1 Behavior of the Relative Wealth

We now examine the role of consumption externalities for the determination of wealth distribution in the steady state. Let us denote the relative capital holding of agent i by $\tilde{k}_i = k_i/K$. Using the capital accumulation equations (18a) and (18b), we derive the dynamics of relative wealth as follows:

$$\dot{\tilde{k}}_t = \frac{1}{K} (f'(K)K - f(K)) (\tilde{k}_i - 1) + \frac{C}{K} \left(\tilde{k}_i - \frac{c_i}{C} \right). \quad (33)$$

García-Peñelosa and Turnovsky (2008) assume that the utility function of each agent is not only identical but also homothetic both from private and social perspective. In this setting individual consumption changes at the same rate so that the relative consumption of each agent, c_i/C , stays constant over time where the level of c_i/C is determined by the initial distribution of wealth among the agents. In contrast, the relative consumption in our model changes during the transition process, which may substantially yield the different outcomes.

For the purpose of comparison, let us first consider the case of identical and homothetic preferences where c_i/C does not change even in the transition process. Observe that the relative wealth along the stable saddle path satisfies the following:¹²

$$\tilde{k}_i(t) = \tilde{k}_i^* + (\tilde{k}_i^* - 1)Z^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}, \quad (34)$$

where

$$Z^* = \frac{(K^* - K(0))\rho A^*}{(\rho - \mu_s)K^*}, \quad (35)$$

$$A^* = 1 + \frac{f''(K^*)K^*}{f'(K^*)} - \frac{K^* f'(K^*)}{f(K^*)} - \frac{\mu_s}{\rho} \left(1 - \frac{K^* f'(K^*)}{f(K^*)} \right).$$

Notice that Z^* and A^* depend only on the steady-state levels of aggregate variables. In this setting, García-Peñelosa and Turnovsky (2008) conclude that the elasticity of substitution between labor and capital in the production function, which affects the sign of A^* , is a key element when determining the wealth distribution in the steady state. To simplify our

¹²See Appendix A with respect to the derivation.

discussion, in what follows we assume that

$$1 \geq \frac{f'(K^*)K^*}{f(K^*)} - \frac{f''(K^*)K^*}{f'(K^*)}, \quad (36)$$

so that A^* in (35) has a positive value. For instance, if $f(K)$ is a Cobb-Douglas production function $f(K) = K^\alpha$ ($\alpha < 1$), then condition (36) is satisfied.

Using (34), we see that the difference of capital stock between the households i and j is:

$$\tilde{k}_i(t) - \tilde{k}_j(t) = (\tilde{k}_i^* - \tilde{k}_j^*) \left(1 + Z^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t} \right). \quad (37)$$

The above expression demonstrates that as long as $K(0) < K^*$, if $\tilde{k}_i^* > (<) \tilde{k}_j^*$, then $\tilde{k}_i(t) > (<) \tilde{k}_j(t)$ for all $t \geq 0$. That is, the catching-up does not arise. The intuitive explanation is as follows. From (5) and (6) $u_1(c_i, C)/u_1(c_j, C)$ is constant over time. Since $u_1(c_i, C)$ monotonically decreases with c_i for all $C (> 0)$, if $c_i(0) > c_j(0)$, then $c_i(t) > c_j(t)$ for all $t > 0$. In view of the intertemporal budget constraint for individual household, the identical preference mean that if $\tilde{k}_i(0) > \tilde{k}_j(0)$, then $c_i(0) > c_j(0)$. As a consequence, if the initial capital distribution satisfies that $\tilde{k}_i(0) > \tilde{k}_j(0)$, then it holds that $\tilde{k}_i^* > \tilde{k}_j^*$ and $c_i^* > c_j^*$. Namely, regardless of the presence of consumption externalities, the initial pattern of wealth distribution is kept in the long run equilibrium.

Now consider the case of heterogeneous preferences. In our general setting, while the relative marginal utility of private consumption, $u_1^i(c_i, C)/u_1^j(c_j, C)$, stays constant over time, c_i/c_j generally changes during the transition. From (5) the relative wealth in our setting is given by

$$\tilde{k}_i(t) = \tilde{k}_i^* + Z_i^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}, \quad (38)$$

where

$$\begin{aligned} Z_i^* &= \frac{B^*(\tilde{k}_i^* - 1)}{K^*} + \frac{\rho - \mu_s}{K^*} \left(1 - \frac{\Phi_i^*}{\Delta^*} \right), \\ B^* &= f''(K^*)K^* + \rho - \mu_s (> 0). \end{aligned}$$

Here, Φ_i^* is given by (23). It is to be noted that the sign of B^* is positive under (36). In view of (38), we find that the difference in capital stock between the households i and j under the

heterogeneous preferences is:

$$\tilde{k}_i(t) - \tilde{k}_j(t) = \tilde{k}_i(0) - \tilde{k}_j(0) + \frac{(Z_i^* - Z_j^*)(e^{\mu_s t} - 1)(K^* - K(0))}{\rho - \mu_s}. \quad (39)$$

Here, the term $(Z_i^* - Z_j^*)$ stems from the presence of preference heterogeneity. As this expression shows, if $Z_i^* < Z_j^*$, then $\tilde{k}_i(0) > \tilde{k}_j(0)$ does not necessarily establish $\tilde{k}_i^* > \tilde{k}_j^*$. If the initially less wealthy household j catches up with the initially richer household i at time \hat{t} , then we can show that

$$\hat{t} = \frac{1}{\mu_s} \log \left(\frac{\tilde{k}_j(0) - \tilde{k}_i(0) + \frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}}{\frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}} \right). \quad (40)$$

The catch-up arises if and only if \hat{t} has a positive value. More specifically, we conclude:

Proposition 4 *Suppose that $K^* > K(0)$ and $k_i(0) > k_j(0)$. Then, (i) an initially poorer household j will never catch up to with an initially richer household if they have an identical preferences; and (ii) if they have different preferences, the initially poorer household will (not) catch up in wealth if the following inequality is satisfied:*

$$\frac{\Phi_j^* - \Phi_i^*}{\Delta^*} > (<) \frac{k_i(0) - k_j(0)}{1 - K(0)/K^*} + \frac{A^*(\tilde{k}_j^* - \tilde{k}_i^*)}{(\rho - \mu_s)}. \quad (41)$$

Proof. Since the catch-up arises if and only if $\hat{t} > 0$, from (40) we can derive the following:

$$0 < \frac{\tilde{k}_j(0) - \tilde{k}_i(0) + \frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}}{\frac{(K^* - K(0))(Z_i^* - Z_j^*)}{\rho - \mu_s}} < 1,$$

which leads to the condition (41) with respect to the catching-up. ■

The condition (41) shows that the catch-up would occur if and only if an initially poorer agent has a sufficiently large degree of preference given by Φ_j^* . This is plausible because the greater elasticity of intertemporal substitution means that the household plans to increase own savings, and hence leads to the greater level of wealth in the future.

We are now interested in how each form of conformism as well as the absolute risk aversion under the subtract and the multiplicative utility functions has the role for the catch-up. For

simplicity, we assume that the catch-up arises just at the steady state, that is, $k_i^* = k_j^*$ and hence, $c_i^* = c_j^*$. Then, we make use of the two specific formulations (11) and (13) discussed in previous sections.

(i) *Subtractive External Effects*

When the utility function is given by $u_i = v_i(c_i - \eta_i(C))$, it holds that

$$\frac{\Phi_j^* - \Phi_i}{\Delta^*} = \frac{\phi_j^*(z_j^*) - \phi_i^*(z_i^*)}{\Delta^*} + \Lambda_j^*(C^*) - \Lambda_i^*(C^*). \quad (42)$$

Note that the catch-up arises at the steady state $c_i^* = c_j^*$; however, because of the heterogeneity of conformism, the steady-state levels of net consumption between agents i and j are not identical $z_i^* \neq z_j^*$. In that case, even if assuming that $v_i(z_i) = v(c_i)$ for all i , we can see that the difference of absolute risk aversion in net consumption, $\phi^*(z_j^*) - \phi^*(z_i^*)$ may play the important role for the catching-up under the small difference in the degree of conformism ($\Lambda_j^*(C^*) - \Lambda_i^*(C^*)$). By contrast, if the two households have similar degree of absolute risk aversion in the steady state, then the main determinant of the sign of $\Phi_j^* - \Phi_i^*$ is the difference in the strength of conformism perceived by households i and j .

(ii) *Multiplicative External Effects*

If the utility function is $u_i = v_i(c_i) \eta_i(C)$, then we obtain:

$$\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_i^*, C^*)}{\Delta^*} = \frac{\phi_j(c_j^*) - \phi_i(c_i^*)}{\Delta^*} + \phi_j^*(c_j^*) \frac{\eta_j'(C^*)}{\eta_j(C^*)} - \phi_i^*(c_i^*) \frac{\eta_i'(C^*)}{\eta_i(C^*)}. \quad (43)$$

Thus if the utility function $v_i(c_i)$ is homogeneous, the condition that the catching-up arises at the steady state can be rewritten as

$$\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_i^*, C^*)}{\Delta^*} = \phi(c_i^*) \left(\frac{\eta_j'(C^*)}{\eta_j(C^*)} - \frac{\eta_i'(C^*)}{\eta_i(C^*)} \right), \quad (44)$$

where note that $\phi(c_i^*) = \phi(c_j^*)$. Namely, if the degree of conformism by the poor is relatively large enough, the catchup may happen. Instead, as long as the heterogeneity of conformism between agents i and j does not exist, the catchup does not arise. Therefore, compared with

the subtract formation of utility function, the role of absolute risk aversion $\phi(c_i^*)$ is limited to some extent.

Finally, we care about the case under which the external effects are common for all agents. Then, the condition (43) can be rewritten as

$$\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_i^*, C^*)}{\Delta^*} = (\phi_j(c_j^*) - \phi_i(c_i^*)) \left(\frac{1}{\Delta^*} + \frac{\eta'(C^*)}{\eta(C^*)} \right),$$

where $1/\Delta^* + \eta'(C^*)/\eta(C^*)$ has a positive sign. Therefore, the necessary conduction for catchup is that $\phi_j(c_j^*) - \phi_i(c_i^*) > 0$. Thus if the positive value of $\frac{\Phi_j(c_j^*, C^*) - \Phi_i(c_i^*, C^*)}{\Delta^*}$ is larger than that of right-hand side of (41), the catchup can hold.

4.2 Patterns of Wealth Distribution

In this subsection we consider the dynamic motion of wealth distribution where $K^* > K(0)$. Defining the difference between the aggregate and the individual capital stocks as $\sigma_i(t) \equiv \tilde{k}_i - 1$, we rewrite (38) as

$$\sigma_i(t) = \sigma_i^* + Z_i^* \frac{K^* - K(0)}{\rho - \mu_s} e^{\mu_s t}. \quad (45a)$$

Again, we first examine the case of identical and homothetic preferences. In this case, from (34), equation (45a) is rewritten as

$$\sigma_i(t) = \sigma_i^* \left(1 + \frac{Z_i^* (K^* - K(0))}{\rho - \mu_s} e^{\mu_s t} \right). \quad (45b)$$

Equation (45b) shows the characteristics of dynamics of relative wealth under the identical wealth. Differentiating (45b) with respect to time yields:

$$\dot{\sigma}_i(t) = \frac{\mu_s \sigma_i^* Z_i^* (K^* - K(0))}{\rho - \mu_s} e^{\mu_s t}.$$

Then, we can see that $\dot{\sigma}_i(t) > (<)0$ if $\sigma_i^* < (>)0$ for all households, thereby being able to guess that the dispersion of wealth shrinks over time under our assumption (36). For example, there is the relative-wealth rich such as $\sigma_i^* > 0$ in the long run. Since $\dot{\sigma}_i(t) < 0$, the divergence between the level of individual wealth and the average wealth decreases over time, and $\sigma_i(t)$ converges $\sigma_i^* (> 0)$, implying that $\sigma_i(0) > \sigma_i^* > 0$. Similarly, if $\sigma_i^* < 0$ in the

long run, the reverse can be applied so that the relative wealth becomes small along time, $0 > \sigma_i^* > \sigma_i(0)$. These results mean that if we define the index of wealth inequality in time t by

$$S = \int_0^1 \sigma_i^2 di,$$

then in the case of identical and homothetic preferences, the steady state level of $S^* = \int_0^1 (\sigma_i^*)^2 di$ is less than its initial level, $S(0) = \int_0^1 \sigma_i(0)^2 di$ as long as (36) holds.¹³

When preferences are heterogeneous, $\sigma_i(t)$ changes according to

$$\dot{\sigma}_i(t) = \frac{\mu_s(K^* - K(0))Z_i^*}{\rho - \mu_s}. \quad (46)$$

In this case, the sign of σ_i^* fails to specify the sign of $\sigma_i(t)$ during the transition. In this situation, we may find the following results:

Proposition 5 *Assume that $K^* > K(0)$, so that the economy is growing during transition under (36).*

- (i) *Suppose that $\sigma_i^* > 0$. (1) If $1 > \Phi_i^*/\Delta^*$, then $\sigma_i(t)$ decreases over time; and (2) if $1 < \Phi_i^*/\Delta^*$, $\sigma_i(t)$ decreases (increases) over time when $Z_i^* > (<)0$.*
- (ii) *Suppose that $\sigma_i^* < 0$. (1) If $\Phi_i^*/\Delta^* > 1$, then $\sigma_i(t)$ increases over time; and (2) if $\Phi_i^*/\Delta^* < 1$, $\sigma_i(t)$ increases (decreases) if $Z_i^* < (>)0$.*

Proof. If $Z_i^* > (<)0$, it holds that $\dot{\sigma}_i(t) < (>)0$ where we use Z_i^* in (38). ■

Intuition behind the results mentioned above is as follows. If all the households have the identical and homothetic preference, the key determinant of dynamic behavior of the relative wealth is the initial level of private consumption selected by each household. Since the initial consumption depends on the initial holding of capital, a wealthier household chooses a higher level of intimal consumption. During the transition it holds that c_i/C stays constant and, hence, the initial divergence between k_i and K continues decreasing under (36).

¹³As can be easily predicted, it holds that $S^* < S(0)$ in the identical preferences:

$$S^* - S(0) = S(0) \left(\frac{-A^*(K^* - K(0))(2(\rho - \mu_s) + A^*(K^* - K(0)))}{(\rho - \mu_s + A^*(K^* - K(0)))^2} \right) (< 0),$$

where we raise both sides of (45b) to the double power and take account of $t = 0$.

By contrast, if preferences are heterogeneous, such a simple pattern of wealth dynamics fails to hold. To see this clearly, it is useful to examine the relative elasticity of intertemporal substitution between the private and the aggregate consumption, Φ_i^*/Δ^* :

$$\frac{\Phi_i^*}{\Delta^*} = \left(1 - \int_0^1 \Lambda_i^* di\right) \left(\frac{\phi_i^*}{\int_0^1 \phi_i^* di} + \frac{\Lambda_i^*}{1 - \int_0^1 \Lambda_i^* di}\right).$$

This expression reveals that the relative elasticity of intertemporal substitution in consumption between the private and the social perspectives is high, if at least one of the following conditions holds: (i) the degree of average absolute risk aversion $\int_0^1 \phi_i^* di$ is small, (ii) the average degree of conformism $\int_0^1 \Lambda_i^* di$ is high; and (iii) the degrees of private absolute risk aversion ϕ_i^* and conformism Λ_i^* , are large. When Φ_i^*/Δ^* takes a relatively high value, the household i attains faster accumulation of her capital than the social average.

Proposition 5 shows that one of the key elements is whether the value of Φ_i^*/Δ^* is greater than the unity or not. In particular, supposing that the agent i is the relative-wealth rich $\sigma_i^* > 0$ under the condition that $1 > \Phi_i^*/\Delta^*$, the initial difference between the average capital and the capital held by the agent i shrinks over time. That is, it holds $\sigma_i(0) > \sigma_i^* > 0$. This is because the degree of elasticity of intertemporal substitution by the agent i is smaller than its average degree, and therefore, the difference between the level of capital stock held by the agent i , k_i and the average level of capital stock, K^* becomes smaller over time. Instead, if $1 < \Phi_i^*/\Delta^*$, we may observe a more complicated relationship. That is, if $1 < \Phi_i^*/\Delta^*$ is large such that $Z_i^* < 0$, then we can see the enlargement of relative wealth $\sigma_i(0) < \sigma_i^*$; alternatively, if $1 < \Phi_i^*/\Delta^*$ but $Z_i^* > 0$, it holds that $\sigma_i(0) > \sigma_i^*$. In a similar way, we can apply for the case (ii) in Proposition 5.

Moreover, let us examine how the formation of conformism affects the results in Proposition 5. For simplicity, we assume that the utility function is in HARA class and the degrees of absolute risk aversions are the same among agents. First, from (12) with $\beta_i = \beta$, we obtain

$$\frac{\Phi_i^*}{\Delta^*} = \left(1 - \int_0^1 \eta'_i(C^*) di\right) \left(\frac{c_i^* - \eta_i(C^*)}{C^* - \int_0^1 \eta_i(C^*)} + \frac{\eta'_i(C^*)}{1 - \int_0^1 \eta'_i(C^*) di}\right). \quad (47)$$

In this case, the relative elasticity of intertemporal substitution is not affected by the wealth distribution. Furthermore, considering that the speed of convergence is also not affected by

the wealth distribution in Proposition 2, whether the society is more unequal or not does not have any impacts on the dynamic motion of relative wealth in (46).

Alternatively, using (14) where $\gamma_i = \gamma$, the relative elasticity Φ_i^*/Δ^* is:

$$\frac{\Phi_i^*}{\Delta^*} = \left(1 - \left(1 - \frac{1}{\gamma}\right) \frac{\int_0^1 \theta_i c_i^* di}{C^*}\right) \left(\frac{c_i^*}{C^*} + \frac{\left(1 - \frac{1}{\gamma}\right) \frac{\theta_i c_i^*}{C^*}}{1 - \left(1 - \frac{1}{\gamma}\right) \frac{\int_0^1 \theta_i c_i^* di}{C^*}}\right), \quad (48)$$

implying that a higher value of $\int_0^1 \theta_i c_i^* di$ yields a higher value of Φ_i^*/Δ^* . That is, supposing that the wealthier agents in the steady state have the stronger degrees of conformism, more unequal society leads to a greater value of Φ_i^*/Δ^* , which means that Z_i^* tends to have a negative sign, and hence $\sigma_i(t)$ tends to increase over time. That is, the capital stock held by the agent i relatively increases compared with the average level of capital stock. Furthermore, considering that the speed of convergence is faster in such a more unequal society from Proposition 3, the value of the dynamic motion of relative wealth in (46) tends to be larger, meaning that the relative wealth $\sigma_i(t)$ largely increases.

We see that the Proposition 5 reveals that the presence of heterogeneous preferences may produce the more-unequal wealth distribution in the steady state. To highlight this fact, define the following:

$$X^* \equiv \left(\frac{\rho - \mu_s}{K^*}\right)^2 \underbrace{\left(1 - \frac{\int_0^1 (\Phi_i^*)^2 di}{\left(\int_0^1 \Phi_i^* di\right)^2}\right)}_{(\#1)} + 2(1 + M^*) \left(\frac{1 - K(0)/K^*}{\Delta^*}\right) \underbrace{\int_0^1 \Phi_i^* \sigma_i^* di}_{(\#2)}, \quad (49)$$

where

$$M \equiv \frac{(1 - K(0)/K^*)B^*}{\rho - \mu_s} (> 0).$$

Using this notation, we may state the following proposition:

Proposition 6 *Assume that, $K^* > K(0)$ under (36). Then in the presence of heterogeneous preferences, it holds that:*

(i) *If $X^* \leq 0$, or if $M^*(2 + M^*)S^* > X^* > 0$, then the long-run wealth inequality is lower than the initial level of inequality.*

(ii) *If $X^* > M^*(2 + M^*)S^*$, then the long-run wealth inequality is larger than the initial level of inequality.*

Proof. From (45a) we can derive the following relation:¹⁴

$$S^* - S(0) = -M^*(2 + M^*)S^* + X^*, \quad (50)$$

which leads to (i) and (ii) where from (50). ■

The intuition in Proposition 6 is as follows. First, let us consider (#1) in (49). Note that this term is always has a negative sign by the Cauchy-Schwarz inequality, which shows the dispersion of degrees of conformism in the entire economy relative to the average degree of conformism. As the negative value of (#1) is greater, X^* has a negative sign, which holds that $S^* < S(0)$.

To understand this effect intuitively, we consider two cases that the dispersion $\int_0^1 (\Phi_i^*)^2 di$ is large and small where the average levels of elasticities of intertemporal substitution are the same $(\Delta^*)^2 = (\int_0^1 (\Phi_i^*) di)^2$. That is, the aggregate capital stocks in two economies converge in the same speeds; instead, the speeds of individual convergence are different. Moreover, supposing that the degrees of absolute risk aversion among agents are homogeneous, we focus on the agents who have the relatively high degrees of conformism in each economy. We can predict that their degrees of conformism in the economy with the large-dispersed conformism are larger in the absolute term than those in the economy with the small-dispersed conformism, which means that their capital stocks converges towards the steady state in a faster speed so that the levels of capital stock are not sufficiently large under the fixed level of aggregate capital stock at the steady state, and hence they do not hold a lot of capital stocks given the level of aggregate capital stock. Instead, in the case of small-dispersed conformism, it takes a longer time under the steady state because the elasticities of intertemporal substitution are not so large. As a result, they hold more capital stocks, thereby leading to a larger dispersion of wealth in the long run.

Second, consider (#2) in (49). This term means that when the agents who have the greater degrees of Φ_i^* have the relatively large levels of capital stock $\sigma_i^* > 0$, the level of X^* becomes larger, and the sign of X^* tends to be positive. As a result, it may hold that $S^* > S(0)$. For example, making use of (12) with $\beta_i = \beta$ and (14) with $\gamma_i = \gamma$, we can show

¹⁴See Appendix B.

that:¹⁵

$$\int_0^1 \Phi_i^* \sigma_i^* di = \frac{\rho K^* S^*}{\beta} - \int_0^1 \frac{\sigma_i \eta_i(C^*)}{\beta} di + \Delta^* \int_0^1 \sigma_i^* \eta_i'(C^*) di, \quad (51a)$$

$$\int_0^1 \Phi_i^* \sigma_i^* di = \frac{\rho K^* S^*}{\gamma} + \Delta^* \left(1 - \frac{1}{\gamma}\right) \left(\int_0^1 \theta_i \sigma_i^* di + \frac{\rho K^*}{C^*} \int_0^1 \theta_i (\sigma_i^*)^2 di \right) \quad (51b)$$

5 Conclusion

In this paper we have studied how the presence of consumption conformism affects the transition dynamics and the stationary wealth of distribution in a neoclassical growth model. In the existing literature on this issue, it has been assumed that households have identical and homothetic preferences, so that wealth distribution will not affect aggregate behavior of the economy. Unlike the existing literature, this paper treats a more general setting where households may have heterogeneous and non-homothetic preferences. Our findings reveal that not only the convergence speed of the economy but also the long-run distribution of wealth are highly sensitive to the strength of conformism. More specifically, a higher degree of conformism in the economy at large accelerates the convergence speed of the economy towards the steady state equilibrium. In addition, the presence of conformism may or may not enhance wealth inequality in the long run. These conclusions suggest that in the presence of consumer conformism, there is ample opportunities of policy intervention that may recover efficiency of resource allocation as well as equality of income and wealth distribution. Since the existing studies on the role of fiscal policy in macroeconomic models with consumption externalities have exclusively used representative-agents models., the effects of policy intervention, in particular tax policies, in the models with conformism and heterogeneities would deserve a further investigation.

¹⁵When $\eta_i(C) = \epsilon_i C$, we can show that $\Phi_i^* = \frac{C^* + \rho K^* \sigma_i^*}{\beta}$, which means that the heterogeneity of conformism does not exist in (49); instead, if $\eta_i(C)$ is non-linear, the difference of conformism affects the wealth inequality at the steady state.

Appendices

Appendix A

We derive the equation (34) and (38) where the derivation is fundamentally the same as García-Peñalosa and Turnovsky (2006, 2008). First, substituting the individual as well as the aggregate capital accumulation equations into $\dot{\tilde{k}}_i = \dot{k}_i/K - \tilde{k}_i\dot{K}/K$ and arranging for it, we can show

$$\dot{\tilde{k}}_i = \frac{1}{K} \left\{ (f'(K)K - f(K))(\tilde{k}_i - 1) + C(\tilde{k}_i - \frac{c_i}{C}) \right\}. \quad (52)$$

Identical preferences: Note that the relative consumption $\frac{c_i}{C}$ is constant over time under the identical preferences. Approximating (52) around the steady state, we can obtain

$$\dot{\tilde{k}}_i = \rho(\tilde{k}_i - \tilde{k}_i^*) + f''(K^*)(\tilde{k}_i^* - 1)(K - K^*) + \frac{\tilde{k}_i^* - c_i^*/C^*}{K^*}(C - C^*), \quad (53)$$

and finally making use of (19a) and (20) and arranging for it, we can derive (34).

Heterogeneous preferences: Since the relative consumption $\frac{c_i}{C}$ is not constant, the linear approximation (52) around the steady state is

$$\dot{\tilde{k}}_i = \rho(\tilde{k}_i - \tilde{k}_i^*) + f''(K^*)(\tilde{k}_i^* - 1)(K - K^*) + \frac{1}{K^*} \left\{ \tilde{k}_i^*(C - C^*) - (c_i - c_i^*) \right\}. \quad (54)$$

Therefore, using $C - C^* = (\rho - \mu_s)(K - K^*)$ and (25), we can show (38) where we use $1 = \frac{\Delta^* f''(K^*)}{\mu_s(\rho - \mu_s)}$ derived by summing (24) over all households.

Appendix B

Raising both sides of (45a) to the double power at $t = 0$ and summing up for all households yields:

$$S(0) = S^* + \frac{2(K^* - K(0))}{\rho - \mu_s} \int_0^1 \sigma_i^* Z_i^* di + \left(\frac{K^* - K(0)}{\rho - \mu_s} \right)^2 \int_0^1 (Z_i^*)^2 di, \quad (B.1)$$

where

$$\int_0^1 Z_i^* \sigma_i^* di = \frac{B^* S^*}{K^*} - \frac{\rho - \mu_s}{\Delta^* K^*} \int_0^1 \Phi_i^* \sigma_i^* di,$$

$$\int_0^1 (Z_i^*)^2 di = \frac{(B^*)^2 S^*}{(K^*)^2} - \frac{2B^*(\rho - \mu_s)}{\Delta^*(K^*)^2} \int_0^1 \Phi_i^* \sigma_i^* di + \left(\frac{\rho - \mu_s}{K^*} \right)^2 \left(-1 + \frac{\int_0^1 (\Phi_i^*)^2 di}{\left(\int_0^1 \Phi_i^* \right)^2} \right).$$

As a consequence, we can show

$$S^* = \frac{S(0) + X^*}{(1 + M^*)^2} (> 0), \tag{B.2}$$

which leads to (50).

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